Metascience (2010) 19:277–280
DOI 10.1007/s11016-010-9363-x

BOOK REVIEW

Mathematical and aesthetic aspects of symmetry

G. Hon, B. R. Goldstein: From summetria to symmetry: the making of a revolutionary scientific concept. Springer, Dordrecht, 2008, xvi + 335 pp, £135.00 HB

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Published online: 11 March 2010

The modern term 'symmetry' brings together a variety of disparate and over-lapping notions, some of which fall under the modern group-theoretic definition now used in mathematical science. The rise of symmetry considerations to dominance in contemporary fundamental physics occurred in the twentieth century, and the modern group-theoretic concept has been explored by philosophers of physics in recent decades. Our scientific concepts are hard-won, forged in the heat of theoretical and practical utility: historical studies displaying some of the ways in which important scientific concepts were formed—and continue to develop—are very welcome.

Hon and Goldstein's book is, primarily, a historical study of the term 'symmetry' and the concepts associated with it up to the early 1800s. They are interested in what becomes of the Greek terms 'summetria' and 'summetros', and the concepts associated with them. Their historical narrative has two aspects: the first traces the use of the term 'symmetry', while the second looks for evidence of features of the modern concept of reflection symmetry independent of terminology.

The historical narrative contains two trajectories, which Hon and Goldstein label 'mathematical' and 'aesthetic'. In mathematics, 'symmetry' was used to mean 'having a common measure', while aesthetically it meant well-proportioned. These 'two trajectories' should not be thought of as capturing a clean separation. The mathematical trajectory is covered in Chapter 2, beginning with Euclid's *Elements*. Hon and Goldstein locate the 'common measure' concept attaching to the term 'symmetry' also in Archimedes (in mathematical contexts, and in mechanics with respect to the balance and equilibrium). Commensurability was an issue in astronomy. Thus, Hon and Goldstein quote from Oresme's discussion of whether the motions of the heavens are commensurable: do the periods of the planets have a common measure?

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The aesthetic trajectory is set out in Chapters 3 and 4. It is here that Hon and Goldstein seek the origins of the modern concept. Their story begins with Plato and Aristotle, but the focus is on Vitruvius. For Vitruvius, 'symmetry' means that the parts stand in an appropriate relation to the whole (they are in proportion). With Alberti, the concept is extended to include 'correspondence' between parts; specifically, identical elements placed at an equal distance on either side of a central element. This notion is found in Perrault's 1673 definition of symmetry which, as others have pointed out, marks a 'decisive move towards the modern concept' (p. 120). Nevertheless, Hon and Goldstein insist (i) on crucial differences between 'the modern concept' and that of Perrault (see below) and (ii) that 'revolutionary' further developments were made by Legendre, whose story is the subject of Chapter 8.

The distinguishing features of Legendre's 1794 definition, according to Hon and Goldstein, are as follows. First, Legendre recognizes what is involved in reflection *explicitly*. Based on the textual evidence offered by Hon and Goldstein, this seems right. However, they make the further claim that Perrault's version does *not* include reflection, even implicitly, and this seems unwarranted.

Second, Legendre's definition drops the relationship of parts to whole and considers instead the relations between two solids: given that they satisfy the requirements of similarity and equality (in magnitude), there remains the question of whether they can be superposed. If they can, they are *absolutely* similar and equal, in Legendre's terminology; if they cannot, they have similarity and equality *by symmetry*. For Hon and Goldstein, the abandonment of part—whole relations is crucial, but to me this is a bit puzzling, since the modern concept crucially retains the notion that parts are inter-changeable with respect to whole.

Third, whereas Perrault's definition concerns the plane, Legendre's concept applies to solids. Indeed, it applies to solids exclusively. Hon and Goldstein emphasize that according to Legendre plane figures are always absolutely similar because you can always flip them in the third dimension. With hindsight, however, we can see that Legendre draws the important conceptual boundary in the wrong place. Just as non-superposable plane figures can be rotated in a third dimension to render them superposable, so for three-dimensional solids rotated in four dimensions: the need for an additional dimension was the crucial conceptual lesson associated with superposability and reflection.

Thus, each 'distinguishing' feature of 'the modern concept' identified in Legendre's definition by Hon and Goldstein seems to me to require further discussion. Hon and Goldstein's conclusion is this (p. 260): 'The difference between Legendre's definition of symmetry in solid geometry, on the one hand, and respective symmetry in architecture, on the other, is categorical.' They point to the increasing use of the term 'symmetry' in scientific texts in the period following Legendre's work as evidence that his definition was a crucial conceptual breakthrough.

Chapter 6, 'The treatment of symmetry in natural history', includes an interesting link between botany and crystallography. Hauy's work makes use of the idea of elements that can undergo substitutions without the crystal changing its appearance. This is very much a part of the modern group-theoretic concept. Hon and Goldstein claim that Hauy's concept of symmetry came from Vitruvius through botany.

Chapter 7 is focused entirely on reflection symmetry, and concerns Euler (on celestial globes) and Kant (on incongruent counterparts).

In addition to the historical narrative outlined above, the book contains a second narrative: a critique of existing historical discussions of symmetry, including the historical stories told by philosophers (such as myself and Elena Castellani). In the *Preface* Hon and Goldstein set out the central thesis of their critical narrative thus: the ‘received view’ is that the modern concept of symmetry has always existed, but in fact prior to the mid-eighteenth century the term ‘symmetry’ does not occur in any of its modern senses and there is no term that has the modern meaning. In what follows I offer some comments on this second narrative, and its impact on the first narrative.

Chapter 5 begins with Copernicus and Galileo, then moves to Kepler, Descartes and Leibniz. Hon and Goldstein show that where the term ‘symmetry’ is used (very rarely) it is in the sense of harmony, or of being well-proportioned. On this basis, they conclude that (p. 176) ‘the usages of symmetry in science and philosophy in the early modern period have nothing to do with the modern concept of symmetry, or with bilateral symmetry and mirror images.’ This conclusion is based on tracing the history of the *term* ‘symmetry’, but the inference seems problematic. For example, Hon and Goldstein point out that Galileo did not use the term ‘symmetry’ in the modern sense. They go on to say: ‘It is often claimed that Galileo appealed to symmetry arguments in the modern sense but, given our historical analysis, this is anachronistic because [the term] symmetry did not have the modern meaning at the time, and Galileo did not use it that way’. But the connection between Galileo and the modern concept of symmetry has nothing to do with his use of the term ‘symmetry’ when discussing the harmony of Copernicus’s system. The reference is to Galileo’s famous ship experiment, which *does* express crucial ingredients of the modern concept: about this there can be no reasonable dispute.

Indeed, for philosophers of physics the exclusive focus on reflection symmetry is frustrating. Reflection symmetry is discrete, but at least as important in contemporary physics are continuous symmetries. These are missing entirely from Hon and Goldstein’s story. For example, their discussion of Newton concerns incommensurability in mathematics, with no mention of the relativity principle, or homogeneity and isotropy of space. Indeed, the methodology adopted by Hon and Goldstein (see Chapter 1) would leave out Einstein’s 1905 paper setting out special relativity, even though this paper is widely regarded as offering the first example of the explicit use of a symmetry principle (the relativity principle) to constrain physical theorizing.

Later in Chapter 5, Hon and Goldstein state that Castellani and I (see Brading, K. and Castellani, E., *Symmetries in Physics*, Cambridge University Press, 2003) claim that Kepler and Leibniz were motivated by symmetry considerations, where symmetry is to be understood in the modern sense. This isn’t right. In the case of Kepler, we are explicitly discussing symmetry understood as harmony. We go on to discuss ‘a different notion of symmetry’ which ‘slowly emerged in the modern era’ (2003, p. 2), and discuss Perrault, but no attempt is made to impute this notion of symmetry to Kepler. Our appeal to the Leibnizian Principle of Sufficient Reason (PSR) occurs in the context of symmetry *arguments*: such arguments describe a situation as exhibiting a certain balance or symmetry, and draw conclusions via

PSR. There is no need to read the modern group-theoretic notion of symmetry (or Leibniz's own PSR) into the historical arguments in order to recognize in them the above general argument form.

I have briefly explained why I disagree with the framework of both the historical and the critical narrative offered by Hon and Goldstein but this will, I hope, be viewed as a contribution to fruitful discussion between historians and philosophers of science. Hon and Goldstein set out their case that one role for history of science is to tell the stories of the emergence of scientific concepts (see Chapter 1). For philosophers of science, such historical work is invaluable.