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A note on rods and clocks in Newton's *Principia*

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1. Introduction

There is an asymmetry in Newton's *Principia* between the status of rods and that of clocks. This can be seen by considering issues highlighted by Harvey Brown in his book *Physical Relativity*, in which he emphasizes the importance of thinking carefully about the relationships between the metrics of space and time, the spatial and temporal behaviors of rods and clocks, and dynamics. Brown's book focuses primarily on Einstein's theories of relativity, but the general lesson applies more widely. In this paper, I examine the status of rods and clocks in Newton's *Principia*. I argue that rods are geometrical whereas clocks are dynamical (section 2), in a sense to be explained, and comment on some aspects of this asymmetry that I find interesting (section 3). I am not suggesting that there is, in fact, an asymmetry in how rods and clocks *should* be treated, nor that Newton thought that there was. On the contrary, Newton sought a dynamical treatment of both rods and clocks. Nevertheless, for the purposes of the *Principia*, rods do not receive a dynamical treatment, do not *need* to receive dynamical treatment, and (given the resources of the *Principia*) would struggle to get such a treatment. The same is not true for clocks.

2. Geometrical rods and dynamical clocks

2.1. Rods and the metric of space

Immediately following the Preface, Newton's *Principia* opens with the following claim (Definition 1):

Quantity of matter is a measure of matter that arises from its density and volume jointly (Newton, 1999, p. 403, p. 403)

Newton says that he means this quantity whenever he uses the term "body" or "mass" (Newton, 1999, p. 404). Thus, the quantity of matter in a body of a given density is measured by its volume. Moreover, for Newton, Place is the part of space that a body occupies (Newton, 1999, p. 409).

Given this, along with Definition 1, it follows that if you divide a quantity of matter of a given density into two equal quantities of matter, each will occupy half of the original region of space: the body fills space uniformly and the spatial characteristics of the body are identical to those of the region of space that it occupies. As a result, there is no possibility of a gap between the measure of space (the length of rods) and the metric of space.

This relationship between the geometrical characteristics of bodies and those of space pre-dates the *Principia*. It is explicit in the manuscript "De Gravitatione" (Newton, 2014), which is an invaluable resource for making the point vivid. In this manuscript, Newton defines place and body as follows:

Definition 1. Place is a part of space which something fills completely.

Definition 2. Body is that which fills place (Newton, 2014, p. 27, p. 27).

He then makes clear what he means by the term "body":

Moreover, since body is here proposed for investigation not in so far as it is a physical substance endowed with sensible qualities, but only in so far as it is extended, mobile, and impenetrable, I have not defined it in a philosophical manner, but abstracting sensible qualities … I have postulated only the properties required for local motion. So that instead of physical bodies you may understand abstract figures in the same way that they are considered by geometers when they assign motion to them … (Newton, 2014, p. 27, p. 27)

Whatever may be the characteristics that we are to abstract from physical bodies, we retain at least the geometrical characteristics of body. Crucially for our purposes, these geometrical characteristics are shared with space. A spherical body, for example, perfectly fills a spherical place in space:

We firmly believe that the space was spherical before the sphere occupied it, so that it could contain the sphere (Newton, 2014, p. 37, p. 37)

There is no gap between the geometrical characteristics of the spherical body and those of the place it occupies. If a ball retains its spherical shape from one place in space to another, then those two places in space are themselves spherical:

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> "we believe all those spaces to be spherical through which any sphere ever passes, being progressively moved from moment to moment"

It is clear that for Newton space is geometrically rich, and that bodies and space share the same geometrical characteristics.[1] Expanding the preceding quotations, we read that:

There are everywhere all kinds of figures, everywhere spheres, cubes, triangles, straight lines, everywhere circular, elliptical, parabolical, and all other kinds of figures, and those of all shapes and sizes, even though they are not disclosed to sight. For the delineation of any material figure is not a new production of that figure with respect to space, but only a corporeal representation of it, so that what was formerly insensible in space now appeared before the senses. For thus we believe all those spaces to be spherical through which any sphere ever passes, being progressively moved from moment to moment … We firmly believe that the space was spherical before the sphere occupied it, so that it could contain the sphere; and hence as there are everywhere spaces that can adequately contain any material sphere, it is clear that space is everywhere spherical. And so of other figures … (Newton, 2014, p. 37, p. 37)

Domski (2013) has argued for a tight connection between the geometrical characteristics of body and space in Newton's philosophy. She highlights Newton's claim that

> "we have an exceptionally clear idea of extension by abstracting the dispositions of a body so that there remains only the uniform and unlimited stretching out of space in length, breadth and depth"

and argues that, for Newton, our knowledge of the geometric form of space *just is* that of our knowledge of the geometric characteristics of bodies, arrived at by abstraction. In short, for Newton, our epistemic access to space is via the extension of bodies, and the spatial characteristics of a body are identical to those of a region of space that it occupies. As a result, as noted above, there is no possibility of a gap between the measure of space (the lengths of rods) and the metric of space.

We understand the reason why there is no possibility of any such gap by recognizing that Newtonian rods are *geometrical* rather than *dynamical*. In *Physical Relativity*, Brown emphasizes the dynamical nature of rods and clocks (as complex physical objects whose behavior is governed by dynamical laws) in relation to the metric of space and time. Physical theory is incomplete until it is able to connect the metric structure postulated in the theory with the behavior of rods and clocks *by means of the dynamics*: the metric structure must be shown to have chronometric significance in terms of the behavior of rods and clocks via the dynamical laws, and not merely by stipulation. With the relationship conceived in this way, there may turn out to be a gap between the lengths of rods and the metric of space, since even the most ideal rod may turn out to measure distances only approximately, and theory may be required to infer from the operational distances to the underlying metric. No such possibility arises for the bodies of the *Principia* because, as I have suggested, the Newtonian rods of the *Principia* are best understood as *geometrical* rather than *dynamical*: Newton's account of body as that which fills place, along with his definition of quantity of matter (connecting mass to volume, and ensuring that body fills place uniformly), yields the result that quantity of matter (or mass) has direct geometrical significance as a measure of space.

Biener and Smeenk (2012) have emphasized that the *Principia* offers two accounts of quantity of matter: the geometrical account found in Definition 1, and the dynamical account found in the Second Law. They show that, for a long time, Newton saw no possibility of conflict between the two, until Cotes pressed him on the issue beginning in 1712. The assumption that masks the potential for conflict is that of extended atoms of *uniform size and mass*. Were this assumption to fail, such that there can be atoms of equal size and different mass, or point particles with finite mass, then the geometrical and dynamical measures of quantity of matter would come apart. The Second Law allows for atoms of equal size and different mass, but Definition 1 does so only on the assumption that variations in density can be taken as primitive. This would have been an unnatural assumption for Newton because it is at odds with the atomist explanation of variations in density (as differences in numbers of atoms per unit volume of a body), and so retaining harmony between the Second Law and Definition 1 comes at a high price. The Second Law also allows for point particles of finite mass, but this is ruled out by Definition 1, according to which all masses have a spatial volume. Not surprisingly, point masses pose a serious problem for the geometrical conception of body, one which the dynamical conception does not face.

It was not until preparations for the second edition of the *Principia* that Newton began to see that the geometrical and dynamical conceptions could come apart. Biener and Smeenk (2012) argue that it is the dynamical conception of quantity of matter that is of more importance in the arguments of the *Principia*. Given the account offered here, of measuring rods as geometrical, were we to give up the geometrical conception of body then we would need a new account of measuring rods, and of their relationship to the metric of space. For example, Newton was (more often than not) an atomist about the microstructure of the material world, and a treatment of measuring rods in terms of atoms and the forces between them invalidates the uniform density of matter at the microlevel (i.e. the claim that body fills place uniformly) central to the relationship between geometrical measuring rods and the metric of space. Newton was, of course, well aware that measuring rods would eventually require a dynamical treatment. However, ridding the *Principia* of geometrical measuring rods turns out to be more difficult that it might at first seem, as we will see in section 3.5, below.

One final point before moving on. I have talked of geometrical rods as measures of space, but Newton distinguished between absolute and relative space. Newton says:

Relative space is any movable measure or dimension of absolute space (Newton, 1999, pp. 408–409)

This seemingly allows for a gap between the metrics of relative space and absolute space, since it seems to leave open the possibility that relative space may be a more or less accurate measure of absolute space. However, Newton also says that:

Absolute and relative space are the same in species and magnitude (Newton, 1999, p. 409, p. 409)

One thing this means is that the metrics of absolute and relative space coincide. For example, if the relative distance between two objects at a given time is 1 m, then that distance is a metre of absolute space too. As a consequence, geometrical rods directly measure both absolute and relative space. The same is not true for clocks and the measurement of absolute and relative time, as we will now see.

2.2. Clocks and the metric of time

Famously, Newton distinguished between "absolute, true and mathematical" time versus "relative, apparent, and common" time

[1] Arthur (1995, p. 332) contrasts this geometrically rich conception of space with that found in Barrow, who "denies that space is an actual existent, or that it has any actual figures, dimensions or parts distinct from those of magnitudes placed in it".

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(Newton, 1999, p. 408), and it is relative, apparent, and common time to which we have epistemic access via bodies in motion.[2]

Relative, apparent, and common time is any sensible and external measure (exact or nonuniform) of duration by means of motion (Newton, 1999, p. 408, p. 408).

The problem with bodies in motion as a measure of time is that the motion of bodies need not be uniform with respect to absolute, true and mathematical time:

It is possible that there is no uniform motion by which time may have an exact measure. All motions can be accelerated and retarded, but the flow of absolute time cannot be changed (Newton, 1999, p. 410, p. 410).

Thus, for Newton, there is the possibility of a gap between the measure of time (the ticking of a clock) and the metric of absolute time.[3]

This is unlike the case of rods where, as we have seen, no such possibility arises. Specifically, whereas the geometrical characteristics of the material rods of the *Principia* cannot but coincide with the geometrical characteristics of absolute space, the "ticking" of the periodic processes of the material clocks of the *Principia* need not coincide with the metrical characteristics of absolute time.

This point can be expressed in terms of absolute and relative space and time. Newton distinguishes absolute and relative space, just as he distinguishes absolute and relative time. However, the metrics of absolute and relative space coincide, and Newton's geometrical rods measure the metric of absolute space. The same is not true for time. Unlike in the case of space, Newton is explicit that the measure of duration by means of motion may be more or less accurate. He clearly states that the metric of relative time may indeed come apart from the metric of absolute time, because the metric of relative time arises from the relative motions of bodies that may be more or less irregular with respect to the metric of absolute time.

The source of this irregularity is in the forces affecting the motions of bodies. It is central to the project of the *Principia* that forces and the motions of bodies are inter-dependent. Newton emphasizes this in the Preface to the first edition:

For the basic problem of philosophy seems to be to discover the forces of nature from the phenomena of motions and then to demonstrate the other phenomena from these forces (Newton, 1999, p. 382, p. 382).

This means that all clocks in the *Principia* are *dynamical* systems, because they are systems of bodies in motion, and motions and forces are inter-related. In other words, their behavior *as clocks* depends on dynamics (in contrast to the case of rods, where a quantity of matter filling space uniformly is introduced independently of dynamics).[4] This is true even for an inertial clock, where the status of the body as an inertial body depends upon the presence – and especially the absence – of forces. In fact, all (or almost all) the clocks in the *Principia* are periodic, involving rotation or oscillation, and so involve forces. The upshot of Newtonian clocks being dynamical is that whether or not there is *in fact* a gap between the ticking of a given clock and the metric of time depends on the details of the dynamics of the clock in question.

I have argued for an asymmetry in the status of rods and clocks in Newton's *Principia*. Specifically, I have argued that rods are geometrical whereas clocks are dynamical, such that there is no possibility of a gap between rods as the measure of space and the metric of absolute space, whereas there is such a gap between clocks as the measure of time and the metric of absolute time. I think that this asymmetry is more interesting and more deeply present within the *Principia* than one might at first suspect, as I aim to show what follows.

3. Aspects of the asymmetry

The asymmetry in the status of rods and clocks in the *Principia*, as geometrical and dynamical respectively, manifests itself in a variety ways. First, it is reflected in the measuring practices needed for the project of the *Principia* (section 3.1). Second, it is a helpful tool in probing some of the similarities and differences in the ways in which Newton treats time and space in the famous scholium on time, space, place and motion. Specifically, where others have seen an unwarranted neglect of absolute time in comparison to absolute space, I see Newton as directing his attention precisely to where it was most needed (section 3.2). Moreover, attention to the asymmetry between rods and clocks enables us to understand the importance of the assertion that absolute time "flows uniformly" (section 3.3). Attention to the dynamical status of certain spatial and temporal concepts more generally is, I think, helpful (section 3.4). With all this said, one might nevertheless harbor the suspicion that the alleged asymmetry between rods and clocks in the *Principia* must be superficial because it is surely readily removed. I think that this is not the case (section 3.5).

3.1. Rods, clocks, and the system of the world

The asymmetry in the status of rods and clocks is reflected in the measuring practices needed for the project of the *Principia*. In order to determine the System of the World (Book III of the *Principia*), two types of observations are needed. First, celestial observations of the stars, and of the positions of the planets with respect to the stars, are required; second, we need to measure the terrestrial acceleration due to gravity, g.

In order to make the celestial observations, we do not need a dynamical account of rods, but we do need to take into account the fact that clocks are dynamical. In order to measure angular distances we make use of the fixed stars, material measuring rods such as sextants, and lines of sight. The distances between the fixed stars are not subject to change due to the forces between them, so these need not be given a dynamical treatment. For the sextant, to the level of accuracy required and the purposes involved, we do not need to consider the forces holding it together or to which it is subjected, and it can be treated as a geometrical object.[5] The lines of sight between the measuring objects and the stars and planets can be treated using geometrical optics; no dynamical treatment is needed. Therefore, our measurements of angular distances do not require a dynamical treatment.

In order to determine inter-planetary distances, the first step makes use of these celestial observations (the positions of the planets relative to the background of the fixed stars), plus trigonometry to calculate the relative diameters of the planetary orbits in terms of the Earth's orbit. Moving from this to an absolute value of distance relies on parallax measurements (which once again use the observed positions of planets relative to the background fixed stars), plus terrestrial distance measurements (geometrical rods) and geometrical calculations. For example, in the 1670s (prior to

[2] For a discussion of the this terminology see Brading (2017).

[3] See Brown, 2005, p. 19.

[4] As noted above, rods will ultimately require a dynamical treatment. However, they are introduced at the outset of the *Principia* as geometrical, and it is of interest to see how far the project of the *Principia* can proceed *without* the need to move to a dynamical treatment of rods. For more on this, see section 3.1, below.

[5] This is an oversimplification, since much technical work was done to improve the stability (and thereby the accuracy) of instrumentation despite changes in conditions such as temperature and humidity.

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the *Principia*) both Giovanni Cassini and John Flamsteed used observations of Mars and different parallax methods in order to determine the Earth-Sun distance. In the course of the development of the *Principia*, the inter-planetary distances become subject to dynamical treatment, through the inverse-square law of gravitation. Nevertheless, the distance measurements that lie at the foundation of the project of the *Principia* rely on measuring rods that can be understood primarily as geometrical rather than as dynamical. The "Phenomena" at the beginning of Book 3 of the *Principia*, from which the argument for universal gravitation begins, are celestial periods and distances. The distances of the satellites of Jupiter and Saturn from their respective planets (Phenomenon 1 and 2) are measured using a micrometer and a telescope (geometrical lines of sight). The mean distances of the planets from the sun (Phenomenon 4) are determined using position measurements with respect to the fixed stars and geometrical calculations.

In contrast, measuring time intervals (including periods of the planets and their satellites) involves appeal to the motions of bodies under forces. More specifically, it involves the motions of a body from our planetary system (such as the rotation of the Earth, the apparent motion of the Sun) and the motion of a pendulum clock. Either explicitly (as in the case of the pendulum) or implicitly (through the equation of time, for example) our clocks require a dynamical treatment, in which we theorize the gap between relative and apparent time versus absolute and true time. In making the positional measurements required for the project of determining the system of the world, we can make do with geometrical rods; but in making the required temporal measurements our clocks must be given a dynamical treatment.

There are issues that need further unpacking if the above claim is to stand. For example, the rotation of the Earth (as used for a clock) was treated as uniform at the time of the *Principia*, and not given a dynamical treatment *qua* clock. However, it was known that in so doing a dynamical assumption was being made. More pressingly, the use of the pendulum clock involves careful treatment of the *length* of the pendulum (as subject to gravity, as well as to temperature changes and so forth), and so a dynamical treatment of rods seems to be involved due to the dynamical status of clocks. Indeed, central to Huygens' treatment of the pendulum clock is the relationship between the length of the pendulum, the time of oscillation, and the acceleration due to gravity (g). This brings us to the second type of observation needed for the project of the *Principia*: that involving terrestrial measurements of g. Huygens' pendulum enabled accurate measurement of g, and this in turn played an important role in Newton's demonstration that the force keeping the Moon in its orbit is the same as that by which terrestrial objects fall to Earth (see Harper, 2011 pp. 31–5). So measurements of distance and time are explicitly entangled with one another in the pendulum clock, and there are further subtleties to be addressed here. These subtleties not withstanding, there remains a rough separation between the geometrical measures of distance and the dynamical measures of duration, as needed for the project of the *Principia*.

As mentioned above, Newton knew that material measuring rods would eventually require a dynamical treatment, and Book 3 of the *Principia* brings dynamics into the measurement of celestial distances. Nevertheless, at the outset of the *Principia* the measurement of space is treated as a geometrical matter, whereas the measurement of time is from the very beginning subject to dynamical considerations. Interestingly, this asymmetry in the treatment of rods and clocks is reflected in the measuring practices used in getting the project of the *Principia* off the ground.

3.2. Arguments for absolute space and time

On the first page of the Introduction to his edited volume *The Concepts of Space and Time* (1976), Milič Čapek points out that Newton's treatment of space and time differ from one another. His interpretation is as follows (1976, p. xv):

> Newton is far more concerned about the empirical status of absolute space than that of absolute time. … Newton tries hard to establish experimentally the difference between the absolute and relative frames of reference (his rotating bucket experiment, and the experiment with the two connected spheres revolving around their common center of gravity),[6] but he does not attempt anything of this sort for time. He candidly concedes – and seems not to be disturbed by it – that no uniform motion, that is no uniform material clock, exists in nature. As we shall see, he was not the first who suspected or explicitly stated it. But what is interesting in the present context is the different lengths with which he treats space and time.

The implication seems to be that Newton should have been more concerned about the empirical status of absolute time, given the non-existence of perfect clocks. Similarly, Gorham (2012, pp. 38–9) claims that the reasons given by Newton in support of absolute time are weaker than those he gives for absolute space. He points out that there is no analogue of the bucket argument (interpreted as offering support for true motion being motion with respect to absolute space). Explicitly echoing Čapek (1976), Gorham writes (p. 38):

> My point is simply that Newton offers no argument of this sort in support of absolute time.

He goes on (p. 38):

> What he offers instead is the observation that 'absolute time, in astronomy, is distinguished from relative time by the equation or correction of the apparent time'.

This is followed by the remark that it is hard to see how this establishes the existence of absolute time.

There is, indeed, an asymmetry in Newton's treatment of absolute time and absolute space, but I think that attention to the differing status of rods and clocks in Newton's *Principia* helps us to see why this is exactly as it should be. For Newton's intended audience, well-versed in the problems of astronomy, the gap between the time parameter in the equation of time, and relative time (as manifest in the periodic motions of observable material bodies), would have been familiar (think especially of Huygens, who used his pendulum clock explicitly in connection with the equation of time). It arises within the *Principia*, as we have seen, due to the dynamical status of clocks in that text. Of course, the abstract time parameter of the equation of time need not be interpreted in terms of Newtonian absolute time; Gorham (2012, pp. 38–9) is quite right that these facts about our practices of time-keeping do not establish the existence of absolute time. However, they do open a gap between the time parameter of the equation of time and any actual motions, and what remains is to show that this time parameter (which is, I repeat, distinct from any relative time) approximates absolute time. Absolute space, on the other hand, and consequently absolute motion, is far harder to establish empirically, precisely because the measure of space opens no gap between the metric of relative space and that of absolute space. On the analysis presented here, this arises from the geometrical status of rods in Newton's *Principia*. Thinking of rods and clocks in this way, as geometrical and dynamical respectively, enables us to see that in focusing his

[6] For discussion of the differing philosophical goals of these two experiments in the context of Newton's arguments in the scholium to the definitions, see Rynasiewicz, 1995a and b.

attention on absolute space Newton directed his attention precisely where it was most needed: demonstrating the need for absolute space as distinct from relative space. Newton's relative "neglect" of time compared to space is no such thing.

The most pressing problem Newton faced concerning space, given the geometrical status of rods, was to distinguish absolute from relative space, thereby making possible the distinction of absolute from relative motion. No such work was needed for absolute and relative time, but this came at the price of generating a new problem, as we will now see.

3.3. *Capricious metrics*

Newton was acutely aware of the gap between the ticking of material clocks and the metric of time. He writes (Newton, 1999, p. 410):

Duration is rightly distinguished from its sensible measures and is gathered from them by means of an astronomical equation. Moreover, the need for using this equation in determining when phenomena occur is proved by experience with a pendulum clock and also by eclipses of the satellites of Jupiter.

Notice that this includes both terrestrial and celestial phenomena. Once this gap between a metric and its measure has been opened up, other things can unravel too. What if the metric of time were capricious with respect to our clocks, so that a tick of a clock could be equal to a unit interval of duration at one time and not equal to it at another time? What if even our most regular material processes were utterly irregular with respect to the metric of time? This would make the metric of absolute time epistemically inaccessible. In my view, this is what Newton's famous claim that time "flows uniformly" rules out: it asserts that time doesn't speed up and slow down relative to material processes that seem to us to be regular. In short, Newton closes the gap by stipulation: he stipulates the time flows uniformly.

Arthur (1995, p. 359) makes the point that Newton saw new significance in the realization that there may be no equable motions corresponding to the time of the astronomers, so that absolute time may have no corresponding measure. The resulting gap between absolute time and its measure has an epistemic correlate, as we have noted, and this epistemic gap, if not the ontological one too, needed to be closed. In his paper, Arthur (1995) demonstrates how and why we should take seriously Newton's claim that time flows. I cannot do justice to this paper here. However, if I have understood Arthur's account correctly, then in the mathematics of Newton's physical theory, the equable flow of time is needed for the continuous generation of physical quantities, and so the regularity (or otherwise) of changes in these quantities with respect to time is built into the mathematics of the theory. For example, the mathematics of fluxions requires that the equable flow of time yields inertial motion, and there is no possibility of a gap:

Since the equable flow of absolute time will correspond with the velocity of a body moving constantly in absolute space, that is, true inertial motion, the idea of equable flow is built into the foundations of Newtonian physics, and gains its warrant from the success of the whole physics of forces applied to the heavens (Arthur, 1995, p. 350, p. 350).

If this is right, then Newton solves the problem of a capricious metric of time not by stipulation, as I have suggested, but by building it into the structure of the dynamical theory. However, as Arthur points out, the mathematics of fluxions turns out to be dispensible. As a result, we are left with a stipulation: to say that time flows equably is to stipulate that material processes that seem to us to be regular are not wildly and erratically irregular with respect to absolute time. After Newton, it was not until Einstein's general theory of relativity that another attempt was made to solve this problem dynamically, rather than by stipulation.

An analogous problem cannot arise for geometrical rods, simply because there is no gap between the geometrical characteristics of material rods and of the spaces that they fill. However, if we change our conception of rods from geometrical to dynamical, then the same problems are going to arise concerning the relationship between the geometry of space and the geometrical properties of rods. What if the metric of space were capricious with respect to our rods, so that what seemed to us to be rods of equal length at different places, in fact occupied different lengths of absolute space?[7] Very late in the drafting of the *Principia*, Newton added to his discussion of absolute space that it is homogenous. He added it in exactly the analogous place to the claim that time flows equably:

Absolute, true and mathematical time, in and of itself and of its own nature, without reference to anything external, flows uniformly and by another name is called duration. …

Absolute space, of its own nature without reference to anything external, always remains homogeneous and immovable.

In my opinion, the stipulation that absolute space is homogeneous does exactly the same work as the stipulation that absolute time flows uniformly: it stipulates the relationship between the measure and the metric, and ensures that the metrics of space and time are empirically accessible via their material measures.

Notice that this solves the problem of how we can know that two temporally separated intervals of time have the same duration, as well has how we can know that two spatially separated intervals of space have the same length. As we have learned (especially from Weyl, see below), the comparison of spatially distant lengths is no more a given than the comparison of temporally distant intervals of time. What we need is a theory that tells us when two distant intervals (be they spatial or temporal) are the same. Transporting a dynamical rod of unit length from one spatial location to another no more guarantees that it has "the same length" at both locations than we have a guarantee that two ticks of a given dynamical clock have the same temporal duration. Whether or not these intervals are equal depends upon the details of the dynamical theory. Newton, it seems to me, had this problem thoroughly under control for the case of clocks, albeit in part by means of stipulation (that time flows equably). It did not arise for rods, because for him rods are not dynamical. It took until the early to mid twentieth century for progress to be made.

The rods and clocks of Einstein's special relativity are geometrical, in exactly the sense described in this paper: there is no possibility of a gap between the measure of spacetime by rods and clocks, and the metric of spacetime. Special relativity is, in this sense, a step backwards from Newton's *Principia*. But in the attempt to move forwards from special relativity, Einstein (and others) moved us beyond Newton. Not only clocks, but rods too, were to be treated dynamically, and the problem of capricious metrics was itself to be solved by dynamics, not by stipulation. This proves to be a tricky undertaking, and a highly delicate matter in general relativity, as Brown's book makes vivid. The image on the front cover of Brown's book is of the waywiser, and the question it is used to raise is this: how does a clock measure duration? There is no analogue of the friction between waywiser wheel and road for a clock and Newtonian space and time, and so nothing to prevent the metric of

[7] One way in which one might try to avoid this is through stipulating the Axiom of Free Mobility, so that a ruler of unit length in one place will, by stipulation, occupy a unit length of space at any other place. The discussions of the Axiom of Free Mobility and its relationship to homogeneous and inhomogeneous spaces, and the reasons why this is a failed attempt to solve the problem, are intimately connected to the issues being raised through our considerations of the status of rods in Newton's *Principia*.

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time being capricious with respect to material processes, and no way for us to rule this out by means of empirical enquiry and physical theorizing. In general relativity, the metric field becomes irreducibly entangled with the gravitational field, to make a single object: the inertial-gravitational field. Matter fields (of which rods and clocks are made) couple dynamically to the inertial-gravitational field. We have the beginnings of the resources for removing the possibility of capricious metrics by appeal to dynamics rather than by stipulation. The complexities in bringing this to fruition are brought into sharp relief by Weyl's 1918 unified theory of gravitation and electromagnetism. In this theory, clocks that initially tick at the same rate may come to tick at different rates when taken through different paths in spacetime (the second clock effect), depending on the presence of an electromagnetic field, and similarly unit measuring rods when brought back together may differ in length. Clearly, this theory involves new complexities in the relationship between the measuring rods and clocks and the spacetime metric, in ways unforeseen prior to Weyl's theory.[8] Brown (2005, pp. 114–118) discusses Weyl's theory as a way of highlighting the importance of such features of general relativity as metric compatibility, and the role that they play in the relationship between the behavior of rods and clocks and the metric of space and time. The central lesson is this (Brown, 2005, p. 160): “The ‘chronogeometric’, or ‘chronometric’, significance of g_{\mu\nu} is not given a priori”. We move beyond the Newtonian stipulations of equably flowing time and homogeneous space when we can answer the following question (Brown, 2005, p. 160):

How does it come about that g_{\mu\nu} is surveyed by rods and clocks, and that its null and time-like geodesics are associated with the world-lines of photons and massive particles respectively? and when we can do so by means of the dynamics of our theory.

3.4. Distance, duration, location and simultaneity

It is often remarked that in the *Principia* the spatial notion of sameness of location is treated as problematic while the corresponding temporal notion of simultaneity is treated as unproblematic. Similarly, it is also remarked that the spatial notion of distance is treated as unproblematic, whereas the corresponding temporal notion of duration is treated as problematic.

Recognition that in the *Principia* rods are geometrical whereas clocks are dynamical offers one way of thinking about why this should be. Since rods are geometrical in the *Principia*, the measure of distance is unproblematic. However clocks are dynamical, and hence the measure of duration becomes an issue to be treated by the details of dynamical theory.[9] In this sense, distance is geometrical whereas duration is dynamical.

It is also straightforward to see that sameness of location is a dynamical notion in the *Principia*. A necessary condition for an object remaining in the same place is that it has no net forces acting upon it. Famously, this is not a sufficient condition, due to Galilean relativity, but from this we see that the question of whether “sameness of location” is empirically accessible becomes an issue to be resolved by attention to the details of the dynamical theory. “Sameness of location” is thus a dynamical notion in the *Principia*.

Simultaneity, on the other hand, lacks any measure, either geometrical or dynamical.[10] Rather, given universal gravitational attraction, absolute simultaneity is presupposed by the form of the laws presented in the *Principia*. Indeed, already in “De Gravitatione” Newton is explicit that.

The moment of duration is the same at Rome and at London, on the earth and on the stars, and throughout all the heavens (Newton, 2014, p. 26, p. 26).

Moreover, in asserting this, and that “we understand any moment of duration to be diffused throughout all spaces” (Newton, 2014, p. 26) Newton does so not to make a point about time, but in order to use this presumed *uncontroversial* point about time in relation to space to illustrate the relationship that God has to space. However, the issue of interest to us is the status of absolute simultaneity within the dynamical theory of the *Principia*. Brown (2005, p. 20) points out that.

Newton spread time through space in inertial frames in such a way that actions-at-a-distance like gravity are instantaneous and do not travel backwards in time in some directions. It is a highly natural convention – it would be barmy to choose any other – but it is a convention nonetheless.

In other words, there is no empirical access to Newtonian simultaneity: it lacks a measure. Nevertheless, as a presupposition of universal gravitation it is probed by the application of the dynamical laws. Brown (2005, p. 20) argues that “Newtonian simultaneity is a by-product of the introduction of forces into the theory” on the grounds that, in a universe consisting entirely of Newtonian free particles, there would be no need for (nor any resources to construct) a privileged notion of simultaneity. However, having introduced universal gravitation as an action-at-a-distance force, a notion of simultaneity must be introduced (be it as a convention, as Brown suggests), and it becomes possible to explore the extent to which the dynamics restricts the possibilities: to what extent does simultaneity turn out to be measurable? This issue has become familiar with Einstein's special theory of relativity.[11]

Returning to the *Principia*, however, the central point is that duration and sameness-of-location are dynamical concepts in the *Principia*, whereas simultaneity and distance are not.

3.5. Dynamical rods and the resources of the Principia

I have argued for an asymmetry between rods and clocks in the *Principia*, with rods as geometrical and clocks as dynamical. One might assume that this asymmetry between rods and clocks is a remnant of Descartes's conception of body as extension, and that it quickly disappears once we adopt a dynamical conception of bodies. One might conclude, therefore, that the asymmetry is a merely superficial feature of the physics of the *Principia*. However, as it turns out, the move from geometrical to dynamical bodies cannot be made quite so quickly or easily as one might hope. The reason is that, surprisingly, the *Principia* lacks the resources for the construction of dynamical rods. This point is emphasized by Stan (2015, pp. 1–5), who delineates three limitations of Newton's laws with respect to extended bodies. For our purposes, the central the problem is that the second law (formulated most familiarly as $F=ma$) is unable to cope with constrained motions, in which the body being acted upon cannot accelerate in the

[8] For a detailed consideration of this issue in Weyl's theory, see Fogel (2008).

[9] In the *Principia*, no question arises as to whether two clocks located at the same place (P), and initially ticking in unison so that they agree on the duration between any two events occurring at P, will also agree on the duration between these two events if one clock is removed from P and then returned to P during the time interval. Such an occurrence would be time dilation, as found in special relativity. There is no hint of this in the *Principia*, but the dynamical status of clocks opens the door to this possibility, as it does to the “second clock effect” (mentioned above, section 3.3).

[10] It is a bit misleading to put it this way, since simultaneity is not a quantity and therefore does not have a measure. However, just as with sameness-of-location, we can examine the extent to which simultaneity is determined by the theory, either geometrically or dynamically, in such a way as to provide empirical access.

[11] Brown (2005, pp. 20–21) draws attention to Poincaré's 1898 essay The *Measure of Time* on this topic.

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direction of the impressed force. Extended bodies, treated as dynamical composites, will consist of constrained particles. Moreover, as a consequence of being constrained, impressed forces may lead not only to linear acceleration of the composite particles on which the force is impressed, but to rotation and (worse) twisting and shearing of the composite object. Stan reports Euler as stating, in 1745, that:

These principles are of no use in the study of motion, unless the bodies are infinitely small, hence the size of a point (Stan, 2015, p. 4, p. 4).

New principles were needed, such as the Torque Law, d'Alembert's principle, the principle of least action, and so forth (see Stan, 2015, pp. 5–9). The 18th century successes of classical mechanics in treating extended bodies, and therefore in making possible a dynamical treatment of rods, were hard won and required resources beyond those of the *Principia*.

Until we can treat extended bodies dynamically, we cannot construct dynamical rods, and the asymmetry between rods and clocks remains. Since this problem proved elusive using only the resources of the *Principia*, it seems to me that the asymmetry between rods and clocks in the *Principia* is not as superficial as it might at first seem.

4. Conclusions

For Newton, in the *Principia*, clocks are dynamical objects whereas rods are geometrical. This asymmetry is connected in interesting ways to other features of his treatment of space and time, and is, I have argued, not so easily removed given the resources of the *Principia*. Nevertheless, one of the lessons of relativity is that remove it we must. In line with the argument of *Physical Relativity*, we must remove it in favor of a dynamical treatment of both rods and clocks.

This note on rods and clocks in Newton's *Principia* began as a talk given in honour of Harvey Brown on the occasion of his 65th year. There was another part to my talk that is not what you find reported here, though this note bears deep and obvious debts to my Ph.D. advisor and friend. At least as important, if not more so, are other lessons that Brown imparts simply from the way that he lives and works: pay attention to details, be humble and charitable towards the work of others, and take good care of your family.

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