Émilie Du Châtelet, *Foundations of Physics*, 1740.
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Footnotes are ours except where otherwise indicated.
Du Châtelet's marginal notes are placed in **{bold}** in the closest appropriate place in the text. Please see the French original for the position of each note in the margin alongside the paragraph. Figures are available in the original text, and online via the BNF.

Chapter 19. Of the Motion of Projectiles

§501. In the two preceding Chapters I have considered only the motion of bodies that fall toward the earth by the force of gravity alone; but when some foreign/outside/external [étrangère] force mixes [se mêle] with its action, as when I throw a stone, then the motion of this stone must necessarily be different from that which it would have had, if it had fallen toward the earth by its own weight alone.

§502. The force that I impress upon the stone that I throw, is called the *projectile force*. This force can be directed perpendicularly or in parallel to the horizon, or indeed it can make any angle with it.

§503. {What is the path of the moveable body when the impelling force is directed perpendicularly to the horizon.} When this force is directed perpendicularly to the horizon, the path of the moveable body is not changed; but its motion toward the earth is only accelerated.

{Or when this force is directed perpendicularly upward.} If this force impels the body along a line that tends perpendicularly upwards, then this body will ascend perpendicularly; but its projectile motion which carries it upwards, will weaken at each instant, and when it has entirely lost it, it will descend toward the earth by the force of gravity, which will then alone be acting on it (§ 319, *num.* 3^0.)

§504. {Why bodies thrown perpendicularly fall back to the same place.} Bodies that are thrown perpendicularly do not, however, fall perpendicularly toward the earth, but fall back describing a curve. For the bodies have already acquired a motion from the rotation of the earth at the moment they are thrown; thus, they fall back toward to the earth by a motion composed of the motion that gravity impresses upon them and of the motion that they have acquired from the rotation of the earth. And thus, this is why they fall back to the same point from which they were projected, although the earth has moved on during the time they have taken to fall.

§505. {What is the path of the moveable body, when the projectile force makes an angle with the horizon.}
If the body is impelled along a line parallel to the horizon, or indeed if this line makes any angle whatsoever with the horizon, then the motion of this body will become a motion composed of the motion that the exterior force acting upon it has communicated to it, and of the motion that gravity impresses upon it at each instant (§. 315, *num*. 1^0.)

§506. The projectile force impressed on the body remains always uniform in a non-resisting medium (§.

[1] Phillip R. Sloan, with Penelope Brading.

315, *num.* 1^0) (and it is in such a medium that I here consider projectile motion). The projectile force remaining therefore always the same, and gravity renewing its action at each instant (§. 315.), the body in obeying these two forces that act upon it at the same time, and one of which is uniform and the other accelerated, will change its direction at every moment; and as a consequence the line that it will describe will be necessarily a curved line (§. 286).

§507. I am going to begin by examining what this curve is in a non-resisting medium when the direction the projectile force is parallel to the horizon.

We have seen in chapter 12 (§. 274) that every body moved by two forces whose directions make between them any angle whatsoever describes, in obeying them, the diagonal of the parallelogram formed by the lines that represent these forces.

Thus, let the body B be thrown in the horizontal direction BR, and let this line BR that represents the projectile force be divided into the equal parts BM, MG, GR **{Plate 10, Fig. 66}**: the body by the force of inertia must traverse in a non-resisting medium equal distances in equal times, in following the projectile motion impressed in the direction BR (§. 234.), since the force which impels it toward BR is assumed to remain always the same; thus, the time of the motion of this body towards point R can be assumed divided as this line into three equal parts; now assuming that in the first moment the projectile force had made the body go from B to M, if it had acted alone upon it, and that during this same time gravity had made it go from B to E, then if its action had been without further interference [sans mêlange], it is clear that the moveable body, in obeying these two forces, will describe in the first moment the diagonal BS of the parallelogram BEMS.

In the second moment, during which the projectile force (which is always the same) would make the body traverse the space ST, equal to BM, gravity would have made it traverse the space SP, three times BE according to the progression of Galileo (§. 305.).

Thus, the body in the second moment, in obeying each of these two forces according to the quantity of each force's action upon it, will describe the diagonal SL of the parallelogram STPL.

Likewise, in the third moment, the space that gravity would make the body traverse, being five times the first, and the projectile force remaining the same, the body will describe the diagonal LD. Now, the diagonals BS, SL and LD joined together do not form a right line; this is because the projectile motion impressed on the body is uniform, or supposed to be such, and the motion impressed by gravity is an equally accelerated motion, and thus the body at each infinitely small instant will approach the center of the earth by an infinitely small diagonal, and all these infinitely small diagonals being joined to one another will form a curve, which turns out to be a semi-parabola.

§508. You have sufficiently studied conic sections to know that one of the properties of the parabola is that the distances along its axis from the origin, and the ordinates of this axis, are among themselves as the squares of these ordinates; thus in the parabola EAC **{Fig. 67}** the distances AP and AM of the axis AR are among themselves as the squares of the ordinates BP and DM.

§509. {The Line that the body describes when it is thrown in a direction oblique or parallel to the horizon is a parabola. ***Fig. 66.*****}** Now is it easy to see that the same properties are found in the curve that the projectiles describe in falling; for the distances BE, BH, BK along the line BK, which represent the spaces traversed by the action of gravity, are among themselves as the squares of the lines ES, HL, KD, which represent the times of fall. For BE is 1, BH is 4, and BK is 9, and ES is 1, HL 2, and KD 3, as a consequence, the line BK can be considered as the axis of the semi-parabola BD, and the lines ES, HL, KD, as the ordinates to this axis. The curve that the projectiles describe in falling toward the earth in a non-resisting medium is therefore a parabola, since it has these properties.

§510. When the direction of the force that threw the body is oblique to the horizon, the curve that it describes is always a parabola, whether the angle formed by the horizon and the line representing this direction is obtuse or acute; for the motion impressed by the projectile force being always uniform in a non-resisting medium, and that of gravity being always equally accelerated in equal times, the curve that results from the combination of these two forces must be the same in all directions, since the forces are the same.

§511. One of the properties of the parabola is furthermore that the parameter of its axis[2] or of one of its diameters[3] is the third proportional to the abscissa of this diameter and its ordinate;[4] that is to say, to the line BE which represents the distance through which the body falls by the action of gravity in the first moment of falling, and the line SE, which represents the distance traversed in the same time by the speed impressed by the projectile force. Thus, since one knows that the distance traversed in the first second by the action of gravity is fifteen feet, then if one knows the distance that the projectile force can make the body traverse in same time of one second, the square of this last distance (which represents the ordinate) being divided by fifteen feet (the distance traversed by the action of gravity, which distance is represented by the abscissa) will give the parameter of the parabola that the body must describe. **{Wolff, *Arithm.* (§. 302)}** Now when one knows the parameter of a parabola, one can describe it: as a consequence, one knows the path of the moveable body, when one knows the distance that the projectile force can make it traverse in a given time, for that which it traverses by the force of gravity is always the same.

It follows from this proposition, that if the projectile motion of two bodies makes them traverse equal distances in equal times, the parabolas which they will describe will have the same parameter.

§512. {Fig. 66.} The line of direction of motion of a projectile is always tangential to the parabola that the body describes. Thus, the line BR touches the parabola BD at point B only. For gravity, acting on the body in the first instant of its motion, changes the direction of this body in this first instant. As a consequence, the line that represents the force impelling this body, being a right line, can touch the curve that this body describes at but a single point.

§513. {Fig. 68.} The parabola BED is called the path of the moveable body, and the right line ST which subtends this parabola BD described by this body in its motion, is called the amplitude of this path, and the angle CBT is called the angle of elevation.

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[2] The parameter of an ellipse is the distance from the focus to a line perpendicular to its axis. *Petit Robert.* This does not quite seem to match with her definition below Ed.

[3] Du Châtelet: One calls the Diameters of a parabola all the lines drawn from the points along the parabola that are parallel to its axis, as in the figure NO [see fig. 67]. The parameter is the line quadruple to the section of the axis contained between the focus and the intersect [foyer & le sommet] of the parabola, and the abscissa is the section of the axis contained between the intersect [sommet] of the parabola and the ordinate to its axis or to one of its diameters.

[4] See definitions in Apollonius, *Conic Sections*, Bk. I, Proposition 15.

§514. {A supposition necessary for the path of the projectile to be a parabola.} In determining that the path of projectiles is a parabola, one was obliged to make several suppositions; for to reduce Physical effects to Mathematical calculations, one is always obliged to suppose many things, and when one then wishes to go back from Mathematical calculations to Physical effects, one finds much loss in exactitude and in precision:

{***Fig. 66***}

1. One supposed that the lines MS, GL, RD that represent the action of gravity on bodies are parallel to one another, for if they were not parallel, the curve described by the body would not be a parabola; but since the action of gravity is always directed towards the center of the earth, the lines MS, GL, RD that represent this action are not parallel, since they would converge at the center of the earth if they were continued.

2. One supposed further that the distances traversed by the projectile force are equal in equal times, but they are not, because of the resistance of the air, which unceasingly diminishes this force and, as a consequence, the distances that it makes them traverse.

3. Finally, one supposed also that the distances traversed by the action of gravity are all in proportion to the square of the times. But this is not exactly true, because this same resistance of the air alters also the proportion of these distances.

§515. The first supposition can be made without sensible error, for the extensions of the greatest projections that we are able to make are so short, in relation to the distance that there is from the surface of the earth to its center, that the differences that result from the lack of parallelism in the lines representing the action of gravity are for us a perfect equality.

{Hist. de l'Acad. 1678.} M. Blondel[5] has calculated that an artillery piece pointed horizontally on a mountain one hundred *toises* high, and that will fire a distance of two thousand, five hundred *toises* when computing the vertical parallel lines, will fire a distance of 2499 *toises*, 5 feet, and 6 ½ *pouces* when computing the change caused by the lack of parallelism in the lines representing the action of gravity, and by whatever inevitable alteration remains in the horizontal line of projection. Now what for us are 5 ½ *pouces* out of 2500 *toises*? This difference is even tinier still in ordinary projections; thus one sees that one can without error count it entirely as nothing.

§516. {In air, the line that projected bodies describe becomes a curve that closely approaches a hyperbola. *Newton, Principia*, Book 2, prop. 4.}
With regard to the resistance of air to vertical and horizontal motion, that one supposes to be zero when one determines that the curve described by falling projectiles is a parabola, its effect is so sensible in the fall of ordinary bodies that the curve that they describe when falling in the air is no longer a parabola, but a curve closely approaching a hyperbola, which receives alterations according to the mass and the form/shape of the bodies, and according to the nature of the air in which they fall.

§517. {The parabola that the projectiles would describe in a non-resisting space, is the foundation of the art of artillery.} Thus, the parabola serves to determine the motion of projectiles in a non-resisting

[5] François Blondel (1618-1686), French engineer and mathematician. Likely reference is Blondel's *L'art de jetter les bombes*, 1683.

medium only, and it is, however, this curve that is the foundation of the art of artillery, for the resistance of air is almost insensible on a body as heavy as a cannon ball, and it is moreover easy in this case to remedy the small irregularities that this resistance can cause.