Émilie Du Châtelet, *Foundations of Physics*, 1740.
Translated by Katherine Brading *et al*.[1] at the University of Notre Dame and Duke University.
Footnotes are ours except where otherwise indicated.
Du Châtelet's marginal notes are placed in **{bold}** in the closest appropriate place in the text. Please see the French original for the position of each note in the margin alongside the paragraph. Figures are available in the original text, and online via the BNF.

Chapter 17. Of rest, and of the fall of Bodies along an inclined plane

400. {The causes by which a Body falling toward earth changes its direction.} The action of gravity is always uniform, and always directed perpendicularly toward the center of the earth (§303 & 338.) Thus, when a body that is falling toward the earth changes its direction or its motion, there must necessarily be some foreign cause that is combined with the action of gravity upon it.

401. These foreign causes can be active or passive; the active causes are those that imprint a new motion on the bodies, such as when I throw a stone that would otherwise have fallen had it been acted upon only by the force of its gravity.

The passive causes are those that do not imprint any new motion on the body, but that change only its direction.

Inclined planes, that is to say, surface planes that make an oblique angle with the horizontal plane, are passive causes that change the direction of the body without imprinting any motion upon it.

402. If these planes were parallel or perpendicular to the horizontal plane, they would not in any way change the direction of the bodies that one had placed there; but in the first case they would present an invincible obstacle to the descent of this body, like the plane AB upon body P, for this body being entirely supported by the plane would remain at rest for all eternity, unless an external cause were to act upon it to disturb its rest.

In the second case, that is to say, if the plane was perpendicular to the horizontal plane as in Figure 38, it would not present any obstacle to the fall of body P, and this body would descend toward the earth along this plane, in the same way as if this plane was not there at all (abstracting from friction) for the action of gravity being always directed perpendicularly to the horizontal plane, the vertical plane AB cannot present any obstacle to its action. **{Fig. 38}**

[1] Penelope Brading, Ashton Green, John Hanson, Lauren LaMore, Mousa Mohammadian, Anne Seul, Phillip Sloan, Monica Solomon.

403. {Inclined planes change the direction of bodies in opposing their fall.} But when this plane is inclined to the horizontal plane, as in Figure 39, then it partially opposes the descent of the body toward the earth. **{Fig. 39}**

Bodies that are falling on an inclined plane therefore have an absolute gravity; and a respective gravity, that is to say, diminished by the resistance of the plane.

Their absolute gravity is the force with which they would descend perpendicularly toward the earth, if nothing was to oppose the motion that carries them there, and their respective gravity is this same force diminished by the resistance of the plane.

The line AC, perpendicular to the horizontal plane, is called the height of the plane. **{Definitions} {Fig. 39}**

404. The line AB, oblique to the horizontal plane, is called the length of the plane.

405. The line BC, that is parallel to the horizontal plane, is called the base of the plane, and the angle ABC, that the plane AB makes with the horizontal plane, is called the angle of inclination of this plane.

406. The respective gravity of a body on an inclined plane is to its absolute gravity as the length of the plane is to the height; for this plane does not oppose the perpendicular descent of the body, and consequently does not diminish its absolute gravity except insofar as it is inclined to the horizontal plane, since if it was perpendicular to it, it would not be opposed to it at all (§401). Therefore, the more this plane is inclined to the horizontal plane, or (which is the same thing) the less high it is, the more the body is supported by the plane, and consequently the less respective gravity it has; therefore the respective gravity of this body on this plane is to its absolute gravity as the height of the plane is to its length.

407. {On an inclined plane, the respective gravity is to the absolute gravity as the height of the plane is to its length.} The respective gravity of the same body on planes that are differently inclined is as the angle of inclination of these planes, for the more this angle increases or decreases, the greater or lesser accordingly is the respective gravity of this body.

Thus, the respective gravity of body P is greater on plane AD than on plan AC, for angle ADB is greater than angle ACB. **{Fig. 40}**

408. If the angle of inclination became a right angle, the respective gravity would coincide with the absolute gravity, to which it would be equal; for then the plane would not resist the fall of the body and so would not diminish its absolute gravity.

409. If this angle vanished, gravity would also vanish, and the body would no longer have any tendency to move along the plane, which would then be horizontal, and if this angle became infinitely small, the respective gravity of this body would become infinitely small.

410. {Of bodies at rest on an inclined plane.} An inclined plane cannot by itself prevent the body that is placed upon it from descending toward the earth, it can only slow its fall: thus, in order for a body to remain at rest on an inclined plane, there must be some force other than the resistance of the plane to support it there.

411. {How a body can be kept in equilibrium on an inclined plane.} {Fig. 41} A body that remains at rest on an inclined plane is kept in equilibrium by two powers that counterbalance its absolute gravity. 1. The resistance of the plane, which acts along the line BD perpendicular to this plane; for the weight P presses on the plane along this line, and so the plane presses on the weight in the same direction, because of the equality of action and reaction. 2. The exterior force that supports this body, on the plane.

412. {What proportion the force that supports the body on an inclined plane must have to the weight in different directions.} The resistance of the plane remains always the same in a same plane, but the direction of the power that supports the body on this plane can change, and this force must be different in its different directions in order to prevent the bodies from falling; for it supports more or supports less in these different directions.

413. {Fig. 42} If the power that supports the body on the plane is vertical like the power SP, it must be equal to the weight of the body; for then it supports the body fully, and the inclined plane no longer counts for anything.

414. **{Fig. 43}** This power will need to be less as its direction moves away from the vertical, so that when this direction becomes parallel to the inclined plane , as in figure 4B, in order for this body P to be supported on the plane AB, the power S must be to the weight of body P as the height of the plane to its length; that is to say, as the respective gravity of this body is to the absolute gravity; for the respective gravity of this body is the only thing that this power has for counterbalancing in this direction.

This direction parallel to the plane is that in which the power that supports the body must be the smallest; for then the resistance of the plane acts in its entirety, and as a consequence the power that prevents the body from falling has that much less to support.

415. As the direction of the power that supports the body increasingly diverges from the parallel to the plane, so this power must be greater in order to prevent the body from falling, such that it must be greater in the direction OP than in the direction SP **{Fig. 44 & 45}**, until finally if it became perpendicular to the plane, as the power KP **{Fig. 45}**, no matter how large it was it could no longer prevent the body from falling along the plane; for it would have only the same action as the plane AB itself, and as a result it could not prevent the body from falling along the plane.

416. Finally, this power could be infinitely small, if the plane was of infinitely small height, which does not need to be proven.

417. {Fig. 46} If the weight L (that I suppose to be the power that supports body P on plane AB), if weight L, I say, rather than keeping body P in equilibrium on plane AB, made it ascend along the length of this plane, while L were itself to descend perpendicularly along line AC, the height to which the weight P will rise will be that to which the weight L will descend, as the height of the plane is to its length. For, supposing that the weight L had raised the weight P from B to R along plane AB, it is as if this weight P had risen perpendicularly the height RH, but the weight L that descends perpendicularly descended the entire height BR; now because of the similar triangles RBH and ABC, RH is to BR as AC is to AB (Euclid, Book 6, Prop. 4). Therefore the height that weight P rose is to the height that body L descended as the height of the plane is to its length, and the heights to which these two weights will rise and will fall will be in inverse ratio to their weights.

418. {Why it is more difficult to go up a mountain than to travel along a plain.} It is easy to see from all that has just been said, why a carriage goes up a mountain with more difficulty than it rolls along a horizontal terrain; for while they are going up the horses must support part of the weight of the carriage, which is to its total weight as the perpendicular height of the plane (that is to say, of the mountain) is to its length. And it is for the same reason that one rolls more easily along a smooth terrain than along a rugged terrain; for the inequalities in the terrain are as small inclined planes.

419. {Fig. 47} Two bodies P and S that are maintained in equilibrium on planes that differ in inclination but are of the same height, are to each other as the length of the planes on which they sit; for they are to each other what the weights would be that would keep them at rest on these planes of which the direction would be parallel to these planes (§414).

420. {On the fall of bodies along an inclined plane.} When no force holds back bodies placed on an inclined plane, they necessarily descend toward the earth along the plane (§410). The motion of the body can then be considered as a composite motion, and the plane on which it descends as the diagonal of the parallelogram formed from the two composing directions **{Fig. 48}**, namely, the perpendicular toward the earth, that gravity imprints at every moment on bodies, and the horizontal caused by the incline of the plane.

421. But this resistance of the plane that impresses the horizontal direction upon the body does not impress any motion upon it, since if it had its effect in its entirety, the body would be at rest; it therefore really only slows the motion that gravity impresses on bodies, and changes the direction of this motion.

422. Thus, in descending along an inclined plane, bodies have no motion other than that which gravity impresses ceaselessly upon them for reaching the center of the earth.

423. Since bodies descend along an inclined plane solely by the force of their gravity, they therefore descend with a uniformly accelerated motion; for the ratio of the respective gravity to the absolute gravity of a body on an inclined plane being always as the height of a plane to its length (§406), and gravity acting always uniformly, the body must move with uniform acceleration in descending along the inclined plane throughout the entire time of the descent.

424. {Bodies follow the same laws in their fall along an inclined plane as in their perpendicular fall.} The descent of weights on an inclined plane therefore follows the same laws as their perpendicular fall. Thus, the spaces that they traverse along an inclined plane are as the squares of their times, or of their speeds; the space that they traverse in accelerated motion is equal to the space that they would traverse in uniform motion during an equal time, and with half the speeds acquired during the acceleration; and finally the spaces traversed in equal, successive times of fall increase as the odd numbers 1, 3, 5, 7, etc. (ch. 13, §306).

425. {But the spaces that they traverse, and the speeds that they acquire, are not equal in equal time.} But if the bodies in their fall along inclined planes follow the same proportions as in their perpendicular fall, the speeds that they acquire and the spaces that they traverse are not equal in equal times to the speeds that they acquire and the spaces that they traverse when they descend perpendicularly.

426. The speed of a body that falls along an inclined plane is the effect of its respective gravity, and its speed in a perpendicular plane is that of its absolute gravity; these speeds must therefore be different, since the causes that produce them are different.

{The speeds along the inclined plane are to the perpendicular speeds in equal time as the height of the plane is to its length.} The speed that the body acquires in falling along an inclined plane is therefore to the speed that it acquires in falling perpendicularly in an equal time, as the height of the plane is to its length, that is to say, like the respective gravity and the absolute gravity that produce these speeds are to each other (§.406), and these speeds conserve the same ratio to each other during all equal times of fall.

427. {Thus, bodies fall more slowly along an inclined plane than along a perpendicular line.} This is why Galileo used an inclined plane to discover the laws of falling bodies; for bodies observe the same proportions in their oblique fall, and in their perpendicular fall, and their oblique fall occurring more slowly, it was easier for him to discern the spaces that the bodies traversed when falling on an inclined plane than when they fell perpendicularly.

428. The spaces that the body traverses in falling along an inclined plane are to those that it would traverse in falling perpendicularly in a determined time, as the speed of the body on an inclined plane is to the perpendicular speed at the end of this time, that is to say, as the height of the plane is to its length.

429. If from the rectangular angle that the perpendicular height of the plane always makes with the horizontal plane, we take a line BD perpendicular to the inclined plane AC, line AD will be to line AB as line AB is to line AC (Euclid, Book 6, Prop. 8) **{Fig. 49}**. Now, we have just seen that the space traversed along the inclined plane is to the perpendicular fall in the same time, as the height of the plane is to its length. The body will therefore traverse along the inclined plane space AD in the same time as that in which it would fall perpendicularly from A to B, since line AD is to line AB as the height of the plane is to its length, and there is along plane AC only this space AD that can be traversed in the same time as space AB, for there is along plane AC only this space AD that can be to space AB as AB is to AC (Euclid, Book 6, Prop. 8).

430. Thus, when we know the space that a body would traverse in falling perpendicularly in a given time, we also know the space that it would traverse in the same time along an inclined plane, of which this perpendicular fall would be the height: from the right angle formed by the vertical and horizontal lines, we draw a line perpendicular to the inclined plane.

431. It is from this proposition that we draw this other one that is in widespread use, namely: *that in a circle of which the diameter is perpendicular to the horizontal plane, the fall of a body along any chord drawn from the ends of the diameter to the circumference will take time equal to that in which a body would traverse the entire diameter.*

{Bodies traverse in equal time all the chords of a circle of which the diameter is perpendicular to the horizontal line.} {Fig.50} In circle ABC, the diameter AB perpendicular to horizontal line LM can be considered as the height of inclined planes AM, AG, where[2] angles ARB, AKB are right angles (Euclid, Book 3, Prop. 31). Thus, lines BK, BR are perpendicular to inclined planes AM, AG, and as a result bodies falling from point A would arrive at the same time at R, K, and B.

[2] We take "or" in the French to be a misprint for "où".

We will prove in the same way that the body must traverse chords KB, RB, in the same time as that in which it would traverse the diameter AB, for one can draw from point A chords AF, AH, equal and parallel to chords RB, KB, and[3] these chords AF, AH will be traversed in the same time as the diameter AB (by §. 431). Therefore chords RB, KB that are equal and parallel to them, will also be traversed in the same time as this diameter AB. **{Fig.50}**

432. It follows obviously from this proposition that the point at which the line taken perpendicularly from the right angle to the inclined plane meets the plane is in the circumference of the circle, whose diameter is the height of the plane.

433. Thus in a circle whose diameter is perpendicular to the horizontal plane, all the chords taken from the ends of this diameter to the circumference are traversed, as well as the diameter itself, in an equal time, and the bodies being left to themselves will arrive at the same time at point B, whether they start from point R, or from point K, or from point O, or from point A, or indeed from any point whatever on the circumference ABC, for each of these chords can be considered as parts of several inclined planes whose height is the diameter AB. **{Fig.55}**

The reason why all of these chords are traversed in equal time is that the more inclined they are,[4] the shorter they are, and the more vertical, the longer they are.

434. The time that a body takes to fall along an inclined plane increases with the inclination of the plane, and this time is to the time of the perpendicular fall as the length of the plane is to its height.

435. Thus, the times of the fall of a body along planes differently inclined but of the same height, are as the lengths of these planes, which does not need proof after what has just been said.

436. {The speeds acquired at the end of a perpendicular fall and of an oblique fall are equal, but the times of these falls are unequal.} I said (in §425) that the speeds acquired along the inclined plane were not equal to the speeds that the body would have acquired in falling perpendicularly during the same time, but what is true for the partial times of the fall, is no longer true for the whole time: for in the parts of the fall, we compare the speeds acquired in the oblique fall during any time to the speeds that the body would acquire in falling perpendicularly during the same time; but in the whole fall, we compare the speeds acquired in the whole time of the two falls, the oblique and the perpendicular. Now, these times are unequal since they are to each other as the length and the height of the plane are to each other. Thus, the speeds of two bodies, one that falls perpendicularly and the other along an inclined plane, would be equal at the

[3] We take "or" in the French to be a misprint for "et".

[4] Inclined towards the horizontal, i.e. less steeply inclined.

end of their fall, even though they were unequal at any time during the fall. **{Fig. 49}** Thus, along the inclined plane ABC, space AB and space AD are traversed in the same time, but the speed that the body acquired at point B and at point D is not equal, the speed acquired at point B is to that which the body acquired at D as AC is to AB, that is to say, as the length of the plane is to its height.

But once the body has arrived at D and continues to fall from D to C, its speed grows in direct proportion to the time of its motion. Thus, the speed acquired at C is to the speed acquired at D as AC is to AD or to AB, that is to say, as the length of the plane is to its height, since the speeds increase as the times (§434). The speeds acquired at B and at C are therefore equal, since they are each to the speed acquired at D as AC is to AB.

This proposition is not among those in which geometry persuades the mind almost in spite of itself; for it is easy to see that the force by which the body tends to fall toward the earth, being all that makes it fall along an inclined plane, when having its whole effect must communicate to the body the same speed, independently of the path along which it falls. Thus, the body has acquired the same speed when it reaches the horizontal plane, whether it arrived along a perpendicular line or along an inclined plane, or along several contiguous inclined planes, so long as it fell from the same perpendicular height.

437. {Fig. 51} It follows from this that a body that has fallen perpendicularly from L to I has acquired the same speed as if it had fallen from H to I; thus, if it continued to fall from I to K along the inclined plane IK, its motion would be the same as if it had fallen from H to K.

But just as its motion is slower along the inclined plane IK than along the perpendicular plane IM (§. 426), a body that falls from L to I and then from I to K would arrive later at the horizontal plane at K than if it had arrived along the perpendicular plane LM, although by either path it would have acquired the same speed; for it used this speed to traverse a longer space in the first case than in the second.

438. Thus, a body descending along the inclined plane LM **{Fig. 52}** will have acquired the same speed at M as if it had fallen from I to M or from Q to G, and if having arrived at M it continued its path the length of the inclined plane MN, it would have the same speed at N as if it had fallen from Q to N or from Q to P; and if having arrived at N it continued its path further, along NO, it would have acquired at O the same speed as if it had fallen from Q to R. **{A curve can be considered as an infinity of contiguous inclined planes.}** Thus, a body that falls along several contiguous inclined planes, such as LM, MN, NO, will have acquired, when it has arrived at the horizontal plane, the same speed as if it had fallen from the perpendicular height of these planes, represented by line QR, assuming that in the changes of direction at M and at N there was not any friction that decreased the speed of the body.

439. {Bodies follow the same laws along curves as along inclined planes.} Since a curve is nothing other than an infinity of infinitely small contiguous inclined planes, bodies descending along curve QH would acquire the same speed as if they had fallen from Q to R.

440. When the angles of inclination of two planes are equal, they are equally inclined, even though their height and their length may be different; for their inclination depends upon the angle that they make with the horizontal plane, and not on their height or their length.

{Fig. 53 & 54} The equally inclined planes ABC, abc having the angle of inclination B and b equal by assumption, and the angle at C and at c being right angles in both cases, these planes form similar triangles whose sides are proportional (Euclid, Book 6, Prop. 4); thus, AB is to ab as AC is to ac. For equally inclined planes, the heights are therefore proportional to the lengths; and if two bodies descend along two planes or along several contiguous planes that are equally inclined, the time that they take to fall along these planes will be for each proportional to the square root of the length of the plane, which does not need proof since these times are always proportional to the square root of the spaces traversed (§315, no. 4).

441. If in place of contiguous planes we imagine two curves composed of infinitely small inclined planes, the times of fall along the two curves will be in the same proportion as along planes that are equally inclined.

442. {Bodies acquire along inclined planes the speed necessary for them to return to the same height from which they have fallen.} It follows from all that has been said in this Chapter, that bodies falling along any surface, whether curved or inclined, acquire the speed necessary to return to the same height, if their direction came to be changed without their speed being diminished, whether they return along the same surface or along some other surface of the same height; for bodies falling along an inclined plane follow the same laws as when they fall perpendicularly. Now in perpendicular fall bodies acquire speeds sufficient for them to return to the same height from which they fell, and these speeds are lost in re-ascending in the same manner as they were impressed upon them in descending, and this is the cause of the oscillation of Pendulums, which I will talk to you about in the next Chapter.