Chapter 3
Structuralist Approaches to Physics: Objects, Models and Modality

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3.1 Introduction

My goal is to develop a structuralist approach to the objects of physics that is realist – but there are obstacles in the way. This paper is about three of them. The first is familiar, having received a great deal of attention in the recent literature, and concerns the suggestion by structural realists French and Ladyman that we should give up talk of objects. This leaves me in the uncomfortable position of being pro-structuralist and pro-realist, but siding with some opponents of structural realism (at least in its ontic form, about which more below) when it comes to objects, so I had better have something to say. In fact I do (see Section 3.4), and I think this obstacle can be moved out of the way. The other two obstacles I have yet to overcome, and the purpose of this paper is to explain what they are, how they arise, and why they are a problem for the structural realist specifically. The resources open to the scientific realist in facing these obstacles are not available to the structural realist, and the reason is the same in both cases.

The latter two difficulties arise in attempting to develop structural realism within the context of the semantic view of theories, and they are challenges that the realist must address in convincing us that his structuralism is indeed a form of realism. I will begin, then, by outlining both a version of the semantic view of theories, modified from the original proposal in three important respects (Section 3.2), and my preferred version of structuralism in this context (Section 3.3). With this in place, I will discuss the first obstacle (concerning structuralism and objects, Section 3.4), and then turn my attention to developing the second and third. These concern the implications of further modifications of the semantic view of theories that are strongly suggested by current physics and by recent work on models (Section 3.5), and the place of modality in structuralist approaches to physics (Section 3.6). As I write, these obstacles stand in the way of my goal of developing a structuralist approach to the objects of physics that is realist. Many of the criticisms that current

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structural realists face are “global”, in the sense that they offer reasons as to why the structural realist project taken as a whole is not viable (e.g. there cannot be relations without relata). My concerns in Sections 3.5 and 3.6 are, in contrast, internal to the project: given that such a project is prima facie viable, what challenges does it face along the way? The obstacles raised in Sections 3.5 and 3.6 fall into this category.

The structural realism of interest here is “ontic” structural realism, as opposed to “epistemic” structural realism, in the following sense. My goal is to develop a structural account of the objects of physics that is complete: no further unknown, or even unknowable, features of objects are part of the content of the theory, or are assumed to be properties of the objects in the world.

3.2 The Semantic View of Theories: Three Modifications

The semantic view of theories arose as an alternative to the syntactic view of theories. According to the syntactic view, a theory consists of two ingredients: an uninterpreted, or partially interpreted, axiom system and correspondence rules or coordinating definitions. These latter are invoked in connecting the theory to the phenomena. The semantic view of theories rejects correspondence rules or coordinating definitions, appealing to “models” instead. Retaining the view that one ingredient of a theory is an axiom system, these models can be understood in the Tarksian sense that they satisfy the axioms of the theory and so are truth-makers (i.e. in each model the sentences of the theory come out true). Moreover, the connection between the theory and the phenomena is made via the models: the models must function not just as Tarskian truth-makers, but also as *representations* of the phenomena.[1]

According to one of the fathers of the semantic view, Pat Suppes, scientific theorizing consists of “a hierarchy of theories and their models” (Suppes 1962, 255; see also Suppes 1960) that bridge the gap between the high level theory and the lower level phenomena (Fig. 3.1). The connection between each level of the hierarchy is made by a relation of *isomorphism* between the models at one level and the models at the next level. An isomorphism is a map that preserves “all the relevant structure”.[2] The idea is that this relationship between the models found at the different levels produces a cascade effect, ensuring that the higher level theory applies to the lower level phenomena.

Recent work – by Steven French and his collaborators concerning the appropriate morphisms, and by Margaret Morrison and others concerning the nature and roles

[1] The term “model” in science is, of course, replete with connotations of representation, and the temptation might perhaps be for users of the semantic view of theories, with its Tarskian models, to slide between models as truth-makers and models as representations without offering a justification. This is not acceptable, but I will not pursue this issue here.

[2] Of course, this is somewhat vague until we say what is meant by “relevant structure”, and that will depend on the case at hand. What can be said in general is that for any given physical theory we will have some handle on what the relevant elements and relations appearing in our models are; the requirement that we can map all these elements at one level of the hierarchy onto the elements at another level, preserving the relations between them, is not trivial.

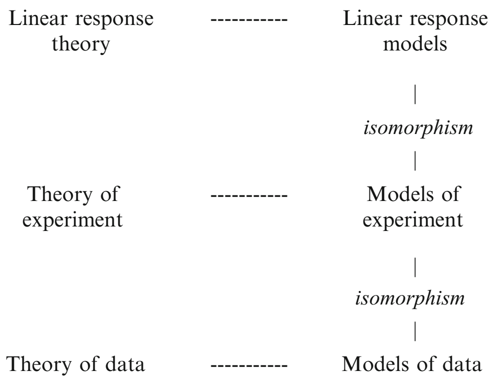

Fig. 3.1 Suppesian possible hierarchy of theories[3]

of models in physics – shows the need for two important modifications to the semantic view of theories. The first modification is the relaxation of the isomorphism requirement such that not *all* the "relevant structure" need be preserved in the mapping from models at one level to models at another level. While French and his collaborators have sought to replace isomorphism with another particular type of morphism (see, for example, Da Costa and French 1990), Elaine Landry and I have argued (2006) that the crucial *philosophical* claim is simply that the models at one level *share structure* with the models at the next level in the hierarchy – we can leave the particular type of morphism, and the type of structures between which we are mapping, open in our general philosophical analysis and investigate it on a case-by-case basis. Making precise the structures and morphisms doing the work may – and in our opinion likely will – depend on features peculiar to a given case. The notion of "shared structure" is weak enough to accommodate this variety, and strong enough to capture the philosophical import of what is common across all cases.

Margaret Morrison (1999), and Morrison and Morgan (1999, Chapter 2), argue persuasively for the partial *autonomy* of "mediating models" in physical theorizing, from both high level theory and from data models. By examining the practice of science they show that there are models which mediate between high level theory and data models, which are not themselves derivable from either the high level theory or the data models. These mediating models share features with both the data models and the high level theory, but are also partially independent or autonomous, this autonomy being crucial to the importance of mediating models in scientific theorizing. I will not rehearse the arguments for this view here: I think they are persuasive and I accept the need for partially autonomous mediating models in our account of scientific theorizing.[4]

[3] Based on Table 1 on p. 259 of Suppes (1962).

[4] One line of response might be to assert that mediating models occur only when the related theory is not mature. I side with those who don't think that this is right, but making my case would require detailed discussions of appropriate case studies. While such discussions lie outside the scope of this paper, I recognize this is an as-yet undischarged responsibility.

Morrison and Morgan (1999, Chapter 2) view the partial autonomy of mediating models as counting against the semantic view of theories and the Suppesian hierarchy, claiming that "models are not situated in the middle of a hierarchical structure between theory and world" (p. 17). However, the relaxation of the isomorphism requirement to one of shared structure allows for the partial autonomy of mediating models from high level theory and from data models whilst retaining mediating models within the hierarchy (see Fig. 3.2, below). Thus, I maintain that mediating models are to be included in the hierarchy, and treated within the (modified) semantic view of theories. (We can note in passing, at this point, that including mediating models turns out to require further modification of the hierarchy with important consequences for structural realism that are, in my opinion, problematic; this will be the subject of detailed discussion in Section 3.5, below.)

There is a further reason why mediating models might be thought to count against the semantic view of theories, and this will bring us to the second modification of the semantic view of theories that I support. According to Morrison and Morgan's account, mediating models are not Tarskian models: there is no theory for which they are truth-makers. Suppes required that at each level of the hierarchy we have a theory, and that the models appearing at that level be Tarskian models of that theory. However, we can again modify the hierarchy to take account of the special character of mediating models by leaving a blank space in the left-hand column of the hierarchy – see Fig. 3.2.

Figure 3.2 shows the two modifications discussed so far. The first modification is that we replace the relationship of isomorphism between the models at the different levels with the more general relationship of shared structure. The second modification is that we insert a level in the hierarchy for mediating models in which

high level theory --- models of high level theory
(Tarskian truth-maker models)
|
relationship of shared structure
|
mediating models
|
relationship of shared structure
|
data models
|
what relationship?
|
"the phenomena" / "the world"

Fig. 3.2 Hierarchy including mediating models there is a blank space to the left of mediating models – there is no theory for which they function as truth-makers.

Note that I have left open how we connect data models to "the phenomena" or "the world". I am in search of a realist account, and therefore one according to which we succeed in representing features of the world. If our high level theory is to have a representational function, then this will be achieved in two steps. First, there is the link between the models of the high level theory and the data models, achieved through the hierarchy via the relationship of shared structure. Second, there is the link between the data models and the world, but in this paper I will not discuss how this crucial step in representation is to be achieved, or how and when we may be justified in asserting that it has been achieved.

The third modification that I advocate, in common with many adherents of the semantic view of theories, is dropping the requirement that a theory be axiomatized, or even axiomatizable in principle. Our concern is physics, in which our high level theories are highly mathematized, and presented in terms of equations of motion, or field equations. At the top of the hierarchy let's put equations and solutions instead of axiomatization and models (see Fig. 3.3, below).[5] We will call the solutions (where generic solutions, in which initial and/or boundary conditions are not specified, are understood as standing for sets of particular solutions)

equations of high level theory --- theoretical models, or generic solutions of the equations

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relationship of shared structure

|

mediating models

|

relationship of shared structure

|

data models

|

what relationship?

|

"the phenomena" / "the world"

Fig. 3.3 Hierarchy in which the axiomatization requirement is dropped

[5] Our theoretical models satisfy our equations, and in this sense might be thought analogous to Tarskian truth-makers. However, we should not take this analogy too seriously, and indeed it has recently been argued that the models of the semantic view are in any case *not* best understood along Tarskian lines. See Thomson-Jones (2006) and Morrison (forthcoming). The account we arrive at may no longer be most aptly termed a "semantic" view of theories, but that is a discussion for another day.

"theoretical models". As an example, consider Newton's laws of motion and his law of universal gravitation. Given these, we can solve the general problem of what orbits are possible for two point masses interacting via the law of gravitation, and indeed using Newton's own argument we can move from point masses to extended spherical bodies of uniform density. Thus, we arrive at the generic solution of the two-body problem. The set of models thus obtained prescribes all and only the possible paths for theoretical Newtonian gravitational objects in two-body motion. These theoretical models are connected to the data models (models constructed from observations of the motions of heavenly bodies) via the hierarchy, in which mediating models serve such purposes as enabling the move from theoretical Newtonian gravitational objects to inhomogeneous extended masses that are not perfectly spherical.

With the semantic view thus modified, we retain the insight that the connection between high level theory and data is to be achieved via *structural relationships* between collections of *models*, these collections forming a *hierarchy*.

3.3 Structuralism and the Semantic View of Theories: Presenting Kinds of Objects

The structuralist finds a natural home within the semantic view of theories, asserting that the relationship of shared structure goes "all the way down" to the phenomena (the empiricist) or to the world (the realist): our theories in physics capture the *structure* of the phenomena or of the world. This needs spelling out in more detail, of course. Here, and in the remainder of this section, I confine my attention to the high level theory and its associated theoretical models. My position is this: the *theoretical kinds of objects* of a high level theory are exhibited in the *shared structure* of the models of that theory. What a theory talks about, at least primarily, is not particular physical objects, but theoretical kinds of objects. We use our theories in physics to talk about physical objects – planets, stars, electrons, atoms, quarks. But our theories are theories of "the electron", or of "G-type stars", or of "the hydrogen atom": what appears in our theories are theoretical *kinds* of objects, not particular objects (such as "*this* electron"). The structuralist view that I advocate claims that the way in which the theory succeeds in talking about these theoretical kinds is via the shared structure of the models of the theory: the shared structure between various models of the theory express the theoretical kinds of objects that are the subject of the theory. With respect to the above example, the claim is that the shared structure of the models constituting the generic solution of the Newtonian equations *present* the *kind* of object "Newtonian inertial-gravitational" for the case of the two-body problem.

I use the term "present" here, rather than "represent", with intent. It is important that we distinguish between theoretical objects and their physical realization. We need to maintain a level of description in which a physical theory can describe electrons, as a theoretical kind, without having to be about electrons as objects that are physically realized in the world: to talk about electrons (or unicorns) is not thereby to bring them into existence as physical objects. I will use the terminology of "presentation" versus "representation" to express this contrast.[6] While a scientific theory *presents* kinds of theoretical objects, it may also seek to *represent* particular physical objects. Indeed, any realist view must surely subscribe to a strong representational role for the theory. But the problems I am concerned with begin even within presentation, and for that reason I am at pains to separate these two roles, and to focus our attention in the first place on presentation. To sum up, the structuralist view I advocate is that the *kinds of objects* that a theory talks about are *presented* via the *shared structure* of the models of the theory.

3.4 Structuralism About Objects

According to the above version of structuralism, theories present kinds of objects. However, French and Ladyman (2003) have argued that we should give up the language of objects and commit ourselves to the ontological primacy of structure.[7] They appeal to two strands of argument, both stemming from "metaphysical individuality". Insofar as I have understood the discussions, the issue of metaphysical individuality concerns whether a particular object can be named such that it may be uniquely re-identified at later times and across possible worlds. There are numerous examples from quantum theory in which the most natural description of the objects involves no commitment to such metaphysical individuality. Paul Teller (1998) has a discussion of this issue where he argues for the superiority, in certain contexts, of the Fock space representation of atomic electrons: we model the electrons in a particular atom using occupation numbers, which are numbers describing how many times each property is instantiated, with no regard to "which" particle has which of the properties (see Teller 1998, p. 128). In other words, we get the *kinds* of electrons, plus the number of electrons instantiating each kind, but no labels enabling us to refer to any one electron in particular – we cannot name the electrons. French and Ladyman's argument is that the interpretation of our theories often fails to *determine* the question of whether the objects involved are individuals or not, and to avoid this "metaphysical underdetermination" (as Ladyman calls it, see his 1998, p. 419) they advocate ontic structural realism because, in committing ourselves to structure rather than to objects, no question can arise over whether the objects of physics are individuals or not.[8]

For French and Ladyman, a realism that commits us to objects but fails to determine the individuality or otherwise of those objects is so strange that they reject it in

[6] See Brading and Landry (2006).

[7] This has led to a lot of noise about whether there can be relations without relata.

[8] Other criticisms of the "argument from underdetermination" are to be found in the literature. See, for example, Chakravartty (2003; 2004, pp. 158–160).

favor of a commitment to “pure structure” as ontologically basic. My view is that individuality is distinct from object-hood, and that the “metaphysical underdetermination worry” over individuality can be avoided in a less dramatic-sounding manner. If a theory makes no commitments concerning whether or not the objects it purports to be about are individuals, then it is inappropriate to conjoin a metaphysics of individuals versus non-individuals to that theory. Indeed, the version of structuralism I have described here talks primarily about *kinds*, rather than particular objects, and as such may not entail any commitments as to whether particular objects instantiating the kind are individuals or not. In such a case, requiring that we discuss these objects in terms of metaphysical individuality (and perhaps even commit ourselves one way or another on the matter) demands that we go beyond the content of the theory: we have to add an interpretational layer not warranted by what the theory itself says.[9] Expressed in this way, the alleged “strangeness” of a commitment to objects that is not accompanied by a metaphysics of individuality doesn’t sound strange at all – at least not to me.

The suggestion is that we can still say there are objects without this entailing that there is a substantive further issue about whether those objects are individuals or not. But in order to talk about such objects we require a *logical* notion of object, at least: we must be able to apply predicates. The philosophical questions about what conditions an object has to satisfy have long been given a double-sided treatment, having both a metaphysical and a logical face (think of Aristotle’s treatment of individuals). On the metaphysical side, an object must be countable[10]; on the logical side it must be capable of serving as an object of predication. According to the view I am advocating, our theories talk about *kinds* of objects. These *kinds* must satisfy the logical condition of serving as objects of predication, but it is a further question whether the physical particulars (the objects) instantiating the kind are logical objects.[11] One point in the debate seems to be the claim that the possibility of *logical* predication depends on appeal to *metaphysically* robust objects – objects that can be named and then re-identified across possible worlds, and over time. However, Simon Saunders (2003a, b) has shown that the logical notion of object, as object of predication, is a weaker notion, requiring only numerical distinctness. He has argued for a version of Leibniz’s Principle of the Identity of Indiscernibles on the basis of which the above example of Fock space poses no problem for the

[9] French and Ladyman (2003) go to strenuous lengths to resist the idea that we can continue talking of objects, simply reconceiving them as nothing more than the relations in which they stand, but I still take it that this is because the objects of physics fail to satisfy conditions which they assume are associated with object-hood. I don’t find a conclusive argument here.

[10] This need not be a sharp criterion for when an object is present.

[11] In other words, I am leaving open the possibility that we can apply predicates to the theoretical kinds of objects without it following that the resulting statements can be re-expressed using a universal quantifier over particular theoretical objects that are the members of the kind. Relatedly, I am not yet convinced that the physical objects instantiating the kind must have theoretical counterparts that are *logical* objects, or that this is necessary in order for us to be able to represent and talk about the world. But I don’t have arguments spelling out these latter points.

logical notion of object, because it admits two-place relations that cope with numerical distinctness for otherwise identical objects: “x is one meter away from y” (for example) gives numerical distinctness by failing to be true when $x = y$ and being true when $x \neq y$. He writes (2003b, p. 294):

> Consider the spherically symmetric singlet state of two indistinguishable fermions. Each has exactly the same mass, charge, and other intrinsic properties, and exactly the same reduced density matrix. Since the spatial part of the state has perfect spherical symmetry, each has exactly the same spatiotemporal properties and relations as well, both in themselves and with respect to everything else. But an irreflexive relation holds between them, so they cannot be identified (namely ‘... has opposite component of spin to...’).

In Saunders’ terminology, they are weakly discernible. Weak discernibility is indeed weak: we cannot refer to one of the two objects in preference to the other. Nevertheless, we can state of the pair that there are two objects, and we can make assertions concerning the properties of those objects. Thus, these objects serve as objects of predication, in the weakened sense given by Saunders’ analysis, enabling Saunders to draw the following conclusion (2003a, p. 131):

> I think they [French and Ladyman] are mistaken in their view that failing transcendental individuality, the very notion of object-hood is undermined by particle indistinguishability in quantum mechanics... It is true that from exact permutation symmetry it follows that such particles... may in certain circumstances not be uniquely identifiable, in the sense that it may not be possible to refer to one member of the collective rather than another. But it does not follow, from logical principles, that such particles cannot be objects of predication. Indeed they can...

In short, on the basis of the metaphysical and logical considerations that French and Ladyman bring to bear, it is not clear to me why we should give up objects and move to “pure structure”. They are right to see interesting metaphysical implications in modern physics for the concept of “object” – we need a weaker notion of “object” than we are used to dealing with. But they are wrong that no such notion is to be had and that we must give up talking about objects and instead talk only about structure.

In the absence of further arguments, I shall assume that the possibility of developing a structural construal of object-hood has not yet been ruled out.

3.5 Realism and the Semantic View of Theories

We come now to an obstacle that I don’t know how to surmount. My purpose is to explain how it arises, and also why it appears to be a problem for the structural realist, while the scientific realist can address the issue with comparative ease. To this end, I begin by discussing a further modification of the semantic view, which leads to “a proliferation of models”. I then discuss the relationship between realism (both scientific and structural) and the semantic view of theories. In the light of the “proliferation of models”, scientific realists have a fairly straightforward response, whereas structural realists face a serious problem.

3.5.1 A Proliferation of Models

Recent work indicates that further modifications to the semantic view of theories, in addition to those described above (see Section 3.2), should be made. Morrison and Morgan (1999) have argued that an important feature of physical theorizing is that *different* mediating models, relating to the *same* theory, may be used for different purposes. Or, transforming their claim into the language of the view that I am advocating: mutually incompatible (collections of) mediating models may share structure with the same collection of theoretical models.[12] Therefore, the hierarchy must be widened as we go down as follows (Fig. 3.4).

The relationship of shared structure accounts for the applicability of high level theory to mediating models and vice versa, and the extent of that applicability.

Morrison (1999) has further shown that there is similar diversity in how data models relate to mediating models, and has argued persuasively that this is necessary to the way that high level theory applies to data models. Once again (as in Section 3.2), I accept Morgan's and Morrison's conclusions, and refer the reader to their work for the supporting arguments. My concern here is to elaborate the consequences of these conclusions for structural realist projects.[13]

In order to take account of this in considering the relationship of mediating models to data models, we must allow that mutually incompatible mediating models (a) may share structure with the same data models, and (b) may not share structure with *all* of the same data models (Fig. 3.5):

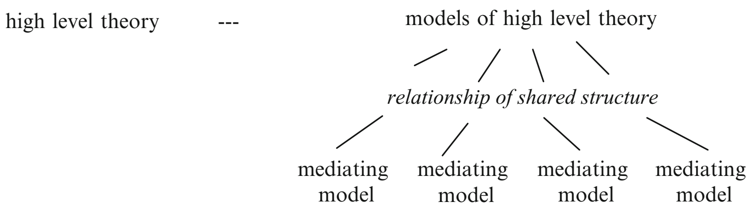

Fig. 3.4 Hierarchy with proliferation of models (I)

[12] Moreover, there may be structure important for certain purposes that is shared by certain mediating models and a *subset* of the theoretical models. What shared structure counts as relevant is dependent on the purpose for which we are using the theory and the models. I will not discuss this further here.

[13] As noted above, French and his collaborators seek to replace isomorphism with a weaker type of morphism (arguing for this in their "partial structures" approach), but reject the generalizing move to "shared structure", where the type of morphism is left unspecified at the general philosophical level and specified only in relation to specific examples. Nevertheless, they too will have to take into account the structural incompatibilities between collections of models within a single hierarchy. I think that the problem I am discussing here affects all structural realism projects once the isomorphism requirement is relaxed (as it needs to be).

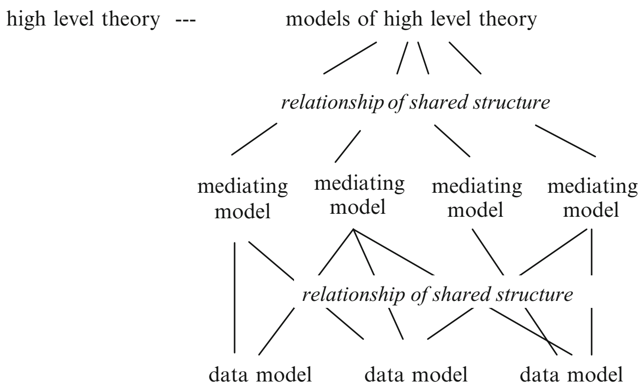

Fig. 3.5 Hiearchy with proliferation of models (II)

This proliferation of models, including incompatible mediating models, poses a serious problem for the structural realist, but not for the scientific realist. In order to see why, we need to compare the commitments made by each.

3.5.2 Scientific Realism

Drawing on van Fraassen (1980, p. 8), we might characterize scientific realism as the endorsement of the following two claims:

Scientific realism

(1) Science aims to give us, in its theories, a literally true story of what the world is like.
(2) We have good reason to believe that science is successful with respect to this aim.

Notice that this characterization of realism makes no reference to the hierarchy of theories. In Suppes' version of the semantic view, each level of the hierarchy consists of a theory and its associated models. However, in our modified version, we have dropped the theory from every level except the top, leaving ourselves with a hierarchy of models from there down. That is, we have the high level theory and its associated models at the top, and then the layers below are occupied by collections of models (mediating models, data models) standing alone. We are therefore faced with a choice concerning the term "theory" in the above characterization of scientific realism: either it refers to the high level theory, appearing at the top of the hierarchy, or it refers to the entire theoretical structure associated with the hierarchy.

The latter interpretation is problematic in the light of the "proliferation of models" discussed above: the models that make up the hierarchy contain mutual incompatibilities, and so some fancy footwork (at least) will be needed in order to extract (preferably in a non ad hoc manner) a logically self-consistent story that could serve as a candidate for a literally true story of what the world is like (as required by the above characterization of scientific realism).

However, if we adopt the former interpretation, then the content of our realist beliefs concerning the nature, make-up and structure of the world is given by the content of the high level theory and its associated models *alone*. Thus, on this view, the problem of mutual inconsistencies raised by the proliferation of models at the lower levels does not infect the content of our realist beliefs. When we are realist about the high level theory "quantum mechanics", and believe that the world indeed contains electrons (say) as described by quantum mechanics, the content of this belief is fixed by the high level theory and its models. The role of the data models and mediating models is to make the link between high level theory and the world, such that we are justified in our claim that our beliefs (expressed using the top level of the hierarchy, i.e. the high level theory and its models) are *about* the world. Expressing things another way, according to this approach the realist who subscribes to the semantic view of theories believes that there is *a model of the high level theory that accurately represents the world* (or some subsystem of it), and the intermediate levels of the hierarchy become "transparent" when it comes to the *content* of the realist's beliefs about the world.[14]

The remaining challenge is to justify the "transparency of the hierarchy". The scientific realist achieves this by appealing to the objects (and their properties) that are the subject of his realist claims. For him, kinds of objects are characterized by the high level theory as a whole. Having been characterized, these kinds of objects then appear in the particular models,[15] and may be traced up and down the hierarchy. For example, the label "electron" attached to a trajectory appearing in a model of the high level theory *means the same thing* as the label "electron" attached to a trajectory appearing in a mediating model or data model: the different levels of the hierarchy deal in the same kinds of objects (as characterized by the high level theory). Herein lies the key to the scientific realist's justification of the "transparency of the hierarchy". The kinds of objects appearing in any model at any level of the hierarchy are labelled *as that kind* from resources *outside* the model (and for mediating and data models, from outside that level of the hierarchy altogether). Therefore, it is legitimate to point to objects in a model of the high level theory and call them electrons (say), and then trace (with further pointing) the presence of these objects (or rather, their trajectories) down through the hierarchy to the data models (and, so the realist hopes, into the world[16]).

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[14] This approach affords a special status to the models associated with the high level theory, and to the high level theory itself. It might be argued that this is a benefit of the approach. See, for example, Morrison (forthcoming), who argues that with the rise of the semantic view of theories and its emphasis on models we risk losing sight of the special role played by theory.

[15] It is compatible with this claim that the high level theory be *identified* with a collection of models, as some advocates of the semantic view of theories maintain.

[16] As noted above (see Section 3.3), representation is carried out in two steps: first the link between the models of the high level theory and the data models; second the link between the data models and the world. I will not discuss the second step here.

The scientific realist accepts that versions of 'the same' object appearing in different mediating models may have incompatible characteristics. Nevertheless, he asserts that these different versions of the same object are understood as different idealizations or approximations – different models – of the same *fundamental kind* of object, where that fundamental kind is characterized by the *high level theory* alone. Thus, he can believe that a model of the high level theory represents the particular physical system he is interested in, and justify that belief in part through appeal to the hierarchy, and nevertheless maintain that the *content* of his belief depends in no way on the lower levels of the hierarchy. "Transparency of the hierarchy", when it comes to our beliefs about electrons (say), is achieved.

To sum up: the challenge posed by the "proliferation of models" can be overcome by the scientific realist as follows. The objects of the scientific realist are characterized by the high level theory, independently of any particular model in which they appear; and the scientific realist can therefore point to electrons (or their manifest traces) whichever model they occur in, without being in any way troubled by this proliferation of models. We are, quite simply, realist about electrons (say), as characterized by our high level theory.

3.5.3 Structural Realism

How does the structural realist fare in the face of the "proliferation of models"? Modifying the above characterization of scientific realism, we can arrive at a version of structural realism in the style of French and Ladyman:

Structural realism

(1) Science aims to give us, in its theories, an accurate representation of the structure of the world.
(2) We have good reason to believe that science is successful with respect to this aim.

A structural realist about a given theory maintains that there is a model of this theory that accurately represents the structure of the world.[17] We are interested in the structural realist who adopts the semantic view of theories, in the revised form that I have discussed. Suppose that this structural realist adopts the same strategy as the scientific realist, so that we interpret "theory" in the above characterization as picking out the top layer of the hierarchy. This leads again to the request for an account of the "transparency of the hierarchy", where here the issue concerns the claim that the *content* of the structural realist's beliefs depends on the structure of the models of the high level theory *only* (while the justification for realism depends in part on the entire hierarchy).

[17] Or, if you prefer, for any sub-system of the world there is a model of the theory that accurately represents the structure of that sub-system.

The structural realist accounts for the "transparency of the hierarchy" in a different way from the scientific realist. In the simplest case, isomorphism between the models at the different levels ensures the sought "transparency" through the "cascade effect", explained above (see Section 3.2): the structure of the theoretical model in question is isomorphic with that of the mediating model, which is in turn isomorphic with that of the data model.

However, according to the version of the semantic view of theories described above (also in Section 3.2), the relationship of isomorphism between the different levels of the hierarchy should be relaxed to one of "shared structure", of which isomorphism is a special case, but which includes weaker relationships. The inclusion of these weaker relationships means that the structural realist must demonstrate the relationship of shared structure on a case-by-case basis, showing that the relevant structure from the high level theoretical model (about which we are supposed to be realist) transfers down the hierarchy appropriately.[18] The task is more arduous than was the case for strict isomorphism, but nothing of principle has changed. When it comes to our beliefs about the structure of the world, the intermediate levels in the hierarchy between the high level theory and the world are rendered transparent by tracing shared structure from the high level theory all the way down the hierarchy: our realist beliefs are expressed in terms of the structure of the high level theory (or its models) alone.[19]

The problem comes when we ask the structural realist to meet the challenge posed by the "proliferation of models". The solution suggested above for the scientific realist is not available to his structural counterpart. The structural realist has been at pains to *reject* objects with the kind of independent characterization that the scientific realist relies upon to overcome the problem of "proliferation of models" and render the hierarchy transparent. The structural realist exhorts us to be realist about the structure of a particular model of our high level theory, claiming that it shares structure with how the world really is. In the face of the proliferation of models, it is unclear how the structural realist can justify the "transparency of the hierarchy" needed for his realist beliefs in the models of the high level theory. Structurally incompatible mediating models, each bearing a relationship of shared structure to a given model of the high level theory, and each being required for the application of that theory to data models, result in a proliferation of incompatible structures at the lowest level of the hierarchy. In the face of this lack of uniqueness, what does it mean to be realist about the structure of a particular model of our high level theory? While the proliferation arises at the level of presentation, it poses a problem for the claim that representation has been achieved: the absence of a unique structure "cascading" down the hierarchy poses a threat to the claim that the structural realist has, in his

[18] Note that this applies equally to the version of structural realism favored by French and his collaborators.

[19] Demonstrating the utility of the notion of shared structure requires the elaboration of detailed case studies, and that will not be carried out here. My concern in this paper is rather with highlighting one of the (problematic) consequences of moving to this more general notion, one that arises as a matter of principle even before the elaboration of case studies.

high level theory, "latched onto" *the* structure of the world.[20, 21] There is a richness in the hierarchy that the scientific realist can accept (or so I have argued), but which threatens to destroy the realism of the structuralist.

In fact, the problem gets worse. McAllister (see especially his 1997) has argued that there is ineliminable human choice in how the data models are structured: there is always an element of freedom in how we separate the data into pattern versus noise. Suppose that we have got as far as making measurements with outcomes plotted as dots on a graph.[22] There is always an element of freedom in how we draw a line through those points – in what counts as a pattern in the dots and what counts as noise. So different data models are compatible with the very same phenomena.[23] The problem that this, and everything discussed in this section, points to is the following: in giving up on isomorphism and allowing a proliferation of structures, it is no longer clear what it means to say that our higher level theory accurately represents *the* structure of the world.[24] And in the absence of this, it is no longer clear that we have anything that might reasonably be called *structural* *realism*.

[20] This is prior to, and independent of, any attempt to give a specific account of representation.

[21] In Section 3.5.1, above, we noted that one might insist that in the characterization of realism the term "theory" applies to the entire hierarchy, rather than to the high level theory alone, thereby avoiding the need to account for the "transparency of the hierarchy". We are now in a position to see why this will not help the structural realist. What could it mean to be realist about incompatible structures?

[22] For now we ignore the enormous amount of work that gets us even to this point, since it is possible to construe this work as discovery of pre-existing quantities.

[23] Separating pattern from noise is not the problem of induction: so construed, it would assume that there is a correct separation (depending perhaps on the completed history of the universe) that – if we hit upon it – would remove the incompatibility between data models. I take McAllister's point to be that the element of human choice is ineliminable *in principle*, not just in practice.

[24] Notice that this problem of different data models being compatible with the same phenomena poses a problem not just for the structural realist but also for the structural empiricist. Paralleling our characterizations of scientific realism and structural realism, we might attempt to characterize structural empiricism as follows:

Structural empiricism

(1) Science aims to give us, in its theories, an accurate representation of the structure of the phenomena.
(2) We have good reason to believe that science is successful with respect to this aim.

I reject both the above realist and empiricist versions of structuralism for two reasons. First, I think that we have good reason to suppose that there is no such thing as *the* structure of the world or *the* structure of the phenomena. (I offer support for this claim with my argument for objects of physics as objects FSPP ("for some practical purposes"), but I will not rehearse this argument here.) Second, I think that set up like this both the realist and the empiricist project are doomed to failure because of the problem of representation – the problem of what justifies us in believing that our theories represent the world or the phenomena. In the realist case, we will have only the "no miracles" intuition to justify our claim that our theories represent the structure of the world, and in the empiricist case it seems that in order to achieve representation van Fraassen (for example) ends up collapsing the distinction between data models and the phenomena, which it seems to me collapses empirical adequacy of a theory with respect to the phenomena into organizational adequacy of the theory with respect to the data models.

3.6 Realism and Modality

The third and final obstacle to structural realism that I will discuss here concerns modality. Ladyman (1998) indicates that the representation of modality plays a constitutive role in making his structuralism a structural *realism*: it "*must* go beyond a correct description of the actual phenomena to the representation of the modal relations between them" (p. 418, my emphasis). In emphasizing modality Ladyman follows Giere (1985), who states that "the crucial dividing line between empiricism and realism" concerns the status of modality, and urges that representing the world as modal is essential to the explanatory resources that the realist wishes to appeal to in explaining the actual. However, I think that there is an obstacle to the structural realist attempt to represent the world as modal. In this section I explain what the problem is, and I offer one doorway that might lead to a solution.

According to the semantic view of theories, a theory successfully represents a physical system if there is a model of the theory that "sufficiently resembles" the system in question. It seems, therefore, a prerequisite for *representing* modal properties of a physical system that these modal properties be captured by the associated model. This sounds banal, but my claim is that the structural realist faces a problem here, one that does not arise for the scientific realist.

In scientific realism the properties of the elements of a model are given by appeal to the wider theory, as discussed above (in Section 3.5), and these can include modal properties (perhaps via the claim that the theory describes "natural kinds", such as electrons (say)). Suppose we have a theoretical model of the hydrogen atom, containing one electron and one proton. We label these elements "electron" and "proton", and the properties of these elements are given not by the model alone, but by the theory as a whole. For example, the theory stipulates the allowed orbits for an electron bound to a proton, thereby ruling on aspects of what is possible and impossible for the electron. The theoretical electron in our model can, if we wish, be described in terms of these modal properties, and there is no immediate obstacle to the claim that modal properties are included in the content of the model.

My argument is that the situation is different in structural realism. Putting the above characterization of successful representation into structural terms, *representation* is achieved by a relationship of *shared structure* between a specified model of the theory and the physical system in question. (This is not a complete account, of course, but it is a necessary condition.) When we ask whether this model contains modal information, a great deal hinges – for the structuralist – on whether by "model" we mean generic or particular model, as I shall now argue.

Very often, when we are modelling a (closed) system, we are seeking to capture how that system is likely to behave given a range of possible initial and boundary conditions. In the example of Newtonian gravitational mechanics, we are seeking a generic solution of our equations (for whatever system is in question). In this sense, our "model" contains modal information: it contains a range of possible trajectories for the system, specifying what the system can and must do, corresponding to a range of initial and boundary conditions. The modal information is carried not by one particular solution/model/trajectory, but by the generic solution/model or collection of trajectories. One option for the structuralist might therefore be to assert that he is realist about the generic solution. But what does this mean? In any given case, only one of the trajectories is in fact realized. We can be structural realist about *this* solution by claiming that it accurately represents the structures and relations associated with the actual trajectory of the physical system in the world. But what does it mean to say that we are realist about the wider structure of the generic solution? In what way are the other, non-realized, trajectories structurally present in the world? I will return to this issue below (see Section 3.6.2), but first I want to pursue the alternative, and interpret "model" not as generic solution, but as particular solution.

3.6.1 Particular Models, and the Place of Modality

Let us assume that the structure that plays a representational role is the structure of *one particular model* of the theory, and not the structure of the generic model or of the wider theory. However (and this is the key point), *this structure, in and of itself, contains no modal information*. In the case of the scientific realist, we have an account of the relationship between the theory and its models that allows the modal commitments of the theory to be present also in the individual models. When it comes to the structural realist, on the other hand, we lack an account of the role of the wider theory, especially with respect to representation; I will discuss this in more detail in what follows. For now, the point is this: unlike the understanding of models used by the scientific realist, within the structuralist conception of a single model *there is no modality* present. And so, trivially, that model by itself cannot *represent* modality in the world. In short, the model describes *only* the *actual* structure of the physical system. We have arrived at something we might dub "structural actualism".

In what follows, I will attempt to clarify the place of modality in a structural actualist account of scientific theories, and thereby determine what is needed to move beyond it to modal structural realism. As introduced above, structural actualism appeals to a single model of the theory. An obvious place to start, therefore, is by looking for an account of what role the rest of the theoretical structure plays. What the structuralist cannot do is implicitly assume that the wider theoretical structure (a) has a representational role, and (b) is appropriate and adequate for the representation of modality. We are owed an explicit account of both these things.

Recall that a theory is characterized by a collection of models. As a result, not all the relations appearing in any given model have the same status with respect to the theory: some relations are part of the *shared structure* of this collection of models, and some are not. We may say that relative to a theory some features of a particular model are "necessary", in that they are part of the shared structure of the collection of models, and the remaining features of that model are "contingent". Thus, by appeal to the theory as a whole, characterized by a collection of its models, we can say more about the relations holding in the specific model of interest – which ones are necessary with respect to the theory, and which ones are contingent. (I will illustrate this with an example within the setting of my preferred version of structuralism; see below.)

I have argued that for the structuralist, modal information is contained in the shared structure of a collection of models, and that we can use this shared structure to talk about a particular model, distinguishing features of the model that are necessary with respect to the theory from those which are contingent. In this sense, the content of our theory allows us to say more about a particular model than is contained in that model. But this does not imply that the structure of any particular model contains modal content and can represent modal properties of a physical system through a relationship of shared structure. On the contrary, to reach that stronger modal realist claim, we would have to add further argument.

Let us be clear about this: if the structuralist is to make representational claims concerning modality, then he owes us either (a) an account of how modality is transferred from the wider theory to a single model, or (b) an alternative account of representation in which the wider theory plays a more direct role (i.e. in which it is not the case that representation is achieved through a relationship of shared structure between an individual model and the physical system in question).

We must first decide whether to attempt a form of modal structural realism – Ladyman's current project – or to follow van Fraassen (1980, pp. 196–203) in maintaining that modality belongs to theory, not to ontology.[25] Before adjudicating on whether to locate modality at the theoretical or the ontological level, I first want to motivate a little further why modal considerations are important. The theories that we have are not complete; we do not have a theory (or set of theories) for which there exists a model (or set of models) that we know to represent the actual world in its entirety. Far from it. Our commitment to a theory guides our actions, it guides our choices about how to act in the face of the unknown course of future events. Our commitment to a theory is equivalent to our placing a restriction on our beliefs about what will and will not happen based on what *theoretically* can, cannot, and must happen. This, I submit, is the main import of the modal features of our theories, and it is one on which the realist and the empiricist can agree. For the purposes of guiding our actions it is sufficient to maintain our modal discourse at the theoretical level, without making any commitment to modality at the ontological level.

My next move is to explain how modality at the theoretical level is captured within the version of structuralism that I advocate, and I will then return to the question of whether structuralists can also turn their modal claims into representational claims.

According to the version of structuralism that I advocate, the shared structure of a collection of models of a theory presents the *kinds of objects* that the

[25] See Ladyman (2000, 2004) for his criticisms of van Fraassen's position on modality.

theory talks about. These kinds stipulate the possible trajectories for objects instantiating the kind. With respect to the theory, the kind gives necessary and sufficient conditions for a particular object to be a member of that kind – these are modal conditions, they stipulate what an object can and must do if it is placed in certain conditions.[26] Recall the Newtonian-gravitational example I used earlier. Given Newton's laws of motion and his law of universal gravitation, we can solve the generic two-body problem. These solutions are models of the theory, and they prescribe all and only the possible paths for Newtonian inertial-gravitational objects in two-body motion. In doing so, they thereby present the kind of object that the theory talks about (viz., "Newtonian inertial-gravitational"). With respect to each particular solution of the equations, there will be features which are due to the objects being instantiations of this kind, and features which are peculiar to that model. The former are necessary with respect to the theory, and the latter contingent, as described above. As this example re-emphasizes, modality is a feature of the *collection* of models of the theory, and not of any particular model of the theory. According to the view that I am advocating, modality is *presented* through the *shared structure* of the models of the theory. Thus, in the first instance, modality concerns presentation and not representation; it is associated with how we present the theoretical kinds of objects that the theory *talks about*, and not with how we represent the particular physical objects that the theory purports to *be about*.[27]

How should the structural realist respond to this account of modality? One option would be to reject Giere's claim that modal realism is central to realism with respect to scientific theories, underwriting the explanations offered by realists, and to offer instead an actualist account, insisting that it is realist. I will not pursue the issue of whether "structural actualism", as I referred to it above, can be construed as a realist position. In what follows I will use the term "structural actualism" neutrally, and will reserve the term "structural realism" to include a commitment to modal realism. The structural actualist can endorse the above account, placing modality at the level of presentation, rather than representation. The emphasis on shared structure and kinds of theoretical object enables him to correctly locate the modal character of structuralism at the theoretical level and to explain in detail what it consists in. Since we don't know what the actual is going to be, we need our theories to be modal. We don't thereby commit ourselves to anything modal in the world, we are not committed to some kind of modal realism. So argues the structural actualist.

One line of argument that a modal realist might offer, in attempting to move from structural actualism to structural realism, is to say that it would be a miracle that this modal guide to action worked if there was no modality in the world. (From

[26]Note that in practice, for the theories that we have and are developing, the specification of necessary and sufficient conditions may not be complete, there may be consistency problems, and so forth.

[27]I follow E. Landry in my use of this terminology.

what I have heard about the approach Ladyman is developing, this is a line of argument he would endorse, but there is nothing in print by him on this at the moment, so far as I know.) But that's a bad inference: the actualist could claim that his commitment to the theory is a commitment to the view that the one of the models of the theory accurately represents the actual world, and so we can use the theory to rule on what the system actually will and will not do by means of what is possible/impossible at the level of theory (counterfactuals belong in the theory). I think the actualist is right that there is nothing miraculous here, and the "no miracles" line of argument is insufficient to move the position from structural actualism (asserted with respect to a particular model of the theory) to structural realism.[28] We are back to structural actualism.

But Ladyman is seeking structural *realism*, so are there any alternatives? On the basis of the discussion I have given here, Ladyman has the following two premises available:

(P1): *An individual model* of a theory *represents* the particular physical system in question through a relationship of shared structure.
(P2): *Modality* is a feature of a *collection of models*, deriving from their shared structure, and is *not* a feature of any individual model.

Our desired conclusion is

(C): Our theories *represent* the modal properties of the world.

Clearly, this argument is invalid. What is needed is a modification of one of the premises. For example, if we could modify (P2) such that modality is a feature of an individual model, then the argument becomes valid. However, my point is that the structural realist cannot assume this to be the case, he must give us an account of how an individual model could come to be imbued with modal content. How best to modify this argument for the representation of modality, and how to justify the modifications, is the third and final obstacle to structural realism discussed in this paper.

3.6.2 Generic Models, and the Place of Modality

I have argued for and endorse (P2), and so my conclusion is that structural realism requires the rejection of (P1): if structuralists are to represent the world as modal, representation must be something more than a relationship of shared structure between a particular model and physical system. Let us return to the alternative that we discussed briefly at the beginning of this section: could we be structural realists

[28] This is not to say that structural realism is an untenable position, when incorporating modal realism, but only that the arguments for structural realism do not get us to such a position: additional arguments relating explicitly to the modal claim need to be supplied.

with respect to the generic model, i.e., with respect to a structure that contains a range of possible trajectories of the system? *Prima facie*, this seems an odd claim: a particular trajectory is realized by a system, that trajectory shares structure with a particular model, and I am realist about that structure (no problem so far); but in addition it is meaningful to assert that the system shares structure with the other non-realized but possible trajectories, and I am realist about this structure too. Because of this oddness, it seems to me that the structural realist is obligated to fill out some details of these additional claims.

The version of structuralism that I have outlined in this paper allows for this possibility, by suggesting a route for moving from structural actualism to structural realism. The structural actualist asserts that there exists a particular model of the theory that shares structure with the physical system in question. The position I have outlined here requires something more of the structural actualist: for a theory (and not just a model) of that system to be acceptable, there must also be shared structure between the physical system and the *collection* of models, so that the objects of the physical system are displayed to behave in accordance with the theoretical kinds (i.e. the kinds given by the shared structure of the collection of models that characterize the theory). The structural actualist, as we have said, answers what it is for an object to be an instance of a given kind *not* by claiming that it somehow *has* all the modal properties expressed by the theoretical kind, but merely by asserting that its actual behaviour is consistent with the prescriptions deriving from the theoretical kind. But a door has also opened for the structural realist: if the *kinds* that the theory presents are also what the theory *represents*, then the way is clear for the objects instantiating a given kind to inherit the modal properties associated with that kind. In other words, this is one way in which representation of a given physical system might draw not just on a particular model, but on the theory as a whole. Clearly, however, this requires that the structuralist provide an account of representation that differs from (P1), and also from the brute claim that the *generic* model represents the structure of actual systems.

My conclusions are as follows. Insofar as Giere is right that modal realism is essential to the explanations that realists seek to offer, the standard approach to structural realism cannot get beyond structural actualism. The view that I advocate indicates a possible escape route, but it is one that is unlikely to be palatable: a great deal of the appeal of structural realism lies in its enchantingly simple claim that an individual model of a theory represents a particular physical system through a relationship of shared structure. Either way, modality is an obstacle to structural realism that is yet to be overcome.

3.7 Concluding Discussion

The above problems associated with (1) the "transparency of the hierarchy" combined with "the proliferation of models", and (2) modality, have both been derived using a characterization of structuralism according to which there is *a* model of the theory that "shares structure" with the physical situation in question. On this approach, structuralism at the theoretical level collapses into relationism at the ontological level: the structure of the model displays the relations obtaining in the physical situation. Call this structural relationism. Structural relationism is compatible with, but not demanded by, my preferred version of structuralism given in Section 3.3. According to this view, structuralism enters first and foremost when discussing not an individual model of the theory but the *shared structure* of the models of the theory. The central claim is that the theoretical kinds of objects that a high level theory talks about are exhibited by the shared structure of the models of that theory. The example that I gave is of Newtonian gravitational physics, where the models (or, in this case, the generic solution of the Newtonian equations, for the two-body problem) present the theoretical kind of object "Newtonian inertial-gravitational" (for the case of the two-body problem). This is a structuralist construal of the *kinds* (perhaps the "natural kinds") that a theory talks about.[29] It is consistent with structural relationism, but other accounts of how a structuralist construal of kinds should be interpreted with respect to our representation of the particular phenomena are possible. This may be a loophole through which, one way or another, it is possible escape from the problems discussed in this paper.[30]

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[29] Chakravartty (2004) has argued for a structuralism grounded in causal properties, in which he emphasizes the distinction between the nature of entities and the structural relations obtaining between those entities. On the view advocated here, the nature of entities (as members of kinds) is characterized in structural terms. Marrying the two approaches, the conclusion would be that it is not merely that our knowledge of the causal properties of entities is structural (knowledge of relations), as Chakravartty argues, but further that the ontological ground of these causal properties is itself structural. Working out the details of how this would go is a task for another day.

[30] How the loophole might work for (2) has been indicated above. In the case of (1), a first step would be to claim that the shared structure of the collection of models characterizing the high level theory gives sufficient conditions for an object to instantiate a kind, but that the necessary conditions need more careful handling in the face of the proliferation of models.

References

Brading, K., and Landry, E., 2006, Scientific Structuralism: Presentation and Representation. *Philosophy of Science* **73**, 571–581.

Chakravartty, A., 2003, The Structuralist Conception of Objects. *Philosophy of Science* **70**, 867–878.

Chakravartty, A., 2004, Structuralism as a Form of Scientific Realism. *International Studies in the Philosophy of Science* **18**, 151–171.

Da Costa, N., and French, S., 1990, The Model-Theoretic Approach in the Philosophy of Science. *Philosophy of Science* **73**, 248–265.

French, S., and Ladyman, J., 2003, Remodelling structural realism: quantum physics and the metaphysics of structure. *Synthese* **136**, 31–56.

Giere, R., 1985, Constructive realism, in P. M. Churchland and C. A. Hooker (eds.), *Images of Science*, 75–98. Chicago: Chicago University Press.

Ladyman, J., 1998, What is structural realism? *Studies in History and Philosophy of Science* **29**, 409–424.

Ladyman, J., 2000, What's Really Wrong with Constructive Empiricism? Van Fraassen and the metaphysics of modality. *British Journal for the Philosophy of Science* **51**, 837–856.

Ladyman, J., 2004, Constructive Empiricism and Modal Metaphysics: A Reply to Monton and van Fraassen. *British Journal for the Philosophy of Science* **55**, 755–765.

McAllister, J., 1997, Phenomena and Patterns in Data Sets. *Erkenntnis* **47**, 217–228.

Morrison, M., 1999, Models as Autonomous Agents, in M. Morgan and M. Morrison (eds.), 1999, 38–65.

Morrison, M., forthcoming, 2007, Where have all the theories gone? to appear in *Philosophy of Science* 74(2);195–228.

Morrison, M., and Morgan, M., eds., 1999, *Models as Mediators: Perspectives on Natural and Social Science*. Cambridge: Cambridge University Press.

Saunders, S., 2003a, Structural Realism, Again' *Synthese*, **136**, 127–133.

Saunders, S., 2003b, Physics and Leibniz's Principles, in K. Brading and E. Castellani (eds.), *Symmetries in Physics: Philosophical Reflections*. Cambridge University Press, 289–307.

Suppes, P., 1960, A Comparison of the Meaning and Uses of Models in Mathematics and the Empirical Sciences. *Synthese* **12**, 287–301.

Suppes, P., 1962, Models of Data. *Logic, Methodology and Philosophy of Science*. Stanford University Press, Stanford.

Teller, P., 1998, Quantum Mechanics and Haecceities, in E. Castellani (ed.), *Interpreting Bodies: Classical and Quantum Objects in Modern Physics.* Princeton University Press, 114–141.

Thomson-Jones, M., 2006, Models and the Semantic View. *Philosophy of Science* **73**, 524–535.

van Fraassen, B., 1980, *The Scientific Image*. Oxford: Oxford University Press.