Emilie Du Châtelet, *Foundations of Physics*, 1740.
Translated by Katherine Brading *et al*.[1] at the University of Notre Dame and Duke University.
Footnotes are ours except where otherwise indicated.
Du Châtelet's marginal notes are placed in **{bold}** in the closest appropriate place in the text. Please see the French original for the position of each note in the margin alongside the paragraph. Figures are available in the original text, and online via the BNF.

Chapter 21: Of the Force of Bodies

557. You saw in the previous chapter that the principle of continuity, founded on that of sufficient reason, does not allow leaps in nature, and that a body cannot pass from one state to another without passing through all the degrees that lie between them. **{A body cannot pass suddenly from motion to rest, nor from rest into motion.}** Thus, by this law a body that is at rest cannot pass suddenly into motion. It must do it successively, and as by gradations, in acquiring one after the other all the degrees of motion that are between rest and the motion that the body must acquire.

558. A body in motion has a certain force that increases as the speed of the body increases, and decreases as the speed decreases. **{Bodies acquire force successively, just as they do speed.}** Therefore since, as we have just seen, a body does not acquire its total speed all at once, but gradually, the force accompanying this speed also passes successively from the pressing cause into the body that it puts in motion.

559. {Two ways to consider the force of bodies.} Thus, there are two ways to consider the force of bodies: first when the force is about to come into being or is still doing so, and second when the force has already come into being in the body, that is to say, when the body is in a state of actual motion.

560. {All pressure produces either a tendency to motion or an infinitely small speed.} While the force is still coming into being, it is the effect of the pressure of an external cause on the body that receives it.
This pressure impresses on the body an element of motion, if it can give way and obey the soliciting cause. But if the body is held back by an insurmountable obstacle that prevents it from acquiring any of the speed and accumulating in itself any of the degrees of force that the cause that acts upon it can give to it, this cause communicates to the body simply a tendency to motion. The force of gravity, when its action is constrained, is of this latter kind.

Everyone agrees that this is the force that makes bodies fall toward the earth. Now a body

[1] Aaron Wells, John Hanson, Penelope Brading, and with Karen Detlefsen and her students.

that is on a table or suspended on a wire is not able to descend toward the earth because the resistance of the table or of the wire prevents this. Nevertheless, the body presses on the table and pulls on the wire, and thereby shows its tendency to motion which cannot have any effect while these insurmountable obstacles oppose it. The pressure due to heavy body is therefore without effect in these two cases. Or rather, the effects it produces, namely the tension of the wire and the pressure on the table, are *non-harmful effects* that do not consume the pressing cause. **{That which we call 'dead force'.}** Thus, the pressing cause loses none of its force because it does not deploy it, but it simply has a tendency to do so, and *this force* resides eternally in the cause without alteration if the obstacles remain always insurmountable. We call this force that the pressing cause deploys without success *dead force*.

561. {On the element of living force.} When we remove the insurmountable obstacle that was preventing the effect of the pressing cause, and give it the freedom to be deployed and to transfer force into the body it presses upon, then the body at once gives way: it no longer returns the pressures of this cause but receives them and accumulates them in itself, and thus these pressures that were nothing but simple efforts, a dead force, become a living force. But this living force is infinitely small, the element of living force, its very beginning, that cannot become a finite living force until it is repeated an infinite number of times, and until it has been accumulated via an infinite number of successive pressures on the body that receives the motion. This infinitely small force, i.e. the element of living force, is the effect of the pressure that was a dead force while the body was still constrained and could not receive motion, and so these two forces – dead force and the element of living force – have the same measure (which is the mass of the body multiplied by the infinitely small speed that the pressure communicates to it at each infinitely small instant); consequently they are usually conflated (and one can do this without error). But I however prefer to distinguish them here, because there is a real difference between them; for in the first case the infinitely small degrees of force are destroyed at every moment, whereas in the second the degrees of force accumulate in the body that receives the motion. **{The measure of the dead force is the product of the mass with the initial speed.}**

562. {The measure of this element of speed is the same as that of dead force.}
When pressure impresses upon the body that yields to it the first degree of force, or the element of living force, this element is proportional to the small space that the pressure makes the body traverse in a given small amount of time, or to the infinitely small speed that the pressure communicates to the body in this small amount of time, and a pressure that would make the same body traverse double the space in the same amount of time would be double (§541). And as this pressure, that produces in the first moment an element of living force when the obstacle yields infinitesimally, is the same as that which produced a dead force when this obstacle did not yield to its work[2], we know the quantity of pressure that an insurmountable obstacle destroys in

[2] We translate "effort" and "efforts" as "work", noting that "work" has a technical meaning in

relation to another pressure to which the obstacle yields infinitesimally in an infinitely small amount of time: we know this by the space that this pressure (acting upon an insurmountable obstacle) would make the obstacle traverse in a given time were the force that it communicated to the body upon which it acts to become living (from the dead that it was previously), as compared to the space that the other pressure (to which the obstacle yields infinitesimally) makes a body of equal mass traverse (considering always the effects in an infinitely small instant).

563. {How we know the work of Machines, and what effects they can produce.} It is in this way that we measure the work done by Machines: by the small spaces the impressed masses would traverse if we allowed them to yield to the work pressing on them, and through examining the relationships these small spaces have to one another.

The force of Machines is a type of dead force, just as is the force of all bodies that tend to actual motion but which are not yet in motion, and to compare them one must calculate their relationship to one another by the product of their mass and their initial speeds which are always proportional to the work that these bodies make in order to move.

{Plate 11, Fig. 71} {Example of the work of Machines like the steelyard.} Thus, let the two arms of a steelyard ME and NE be loaded at their ends with two weights M and N, that hold it in equilibrium.

The relation between these forces can be known if we consider what would happen if one of these arms yielded to the work of the body pressing on it. One sees that the arm ME would go to mE and the arm NE to nE; and that as a consequence body M would describe the small arc Mm while body N would describe the small arc Nn in the same time. The work of each will therefore be as the small spaces Mm and Nn multiplied by their masses. For, these small spaces are as their initial speed. But we assume the work to be equal. Thus mass M is to mass N as space Nn is to space Mm; that is, the masses are in inverse ratio to the spaces, by proposition 16 of book 6 of Euclid. But as the triangles MmE and NnE are similar, their sides are proportional (Euclid, prop. 4, book 6). Thus, Nn:Mm = NE:ME; that is, the spaces traversed are among themselves as the lengths of the arms of the steelyard. Replacing, then, the ratio of the small spaces Nn to Mm with the ratio of the length of arms NE to ME, these being equal, one will have M:N = NE:ME. That is, weights M and N are in reciprocal ratio to the lengths of the arms of the steelyard; this is the fundamental proposition of Statics.

564. {Example of the fundamental proposition of Hydrostatics.} The fundamental proposition of hydrostatics can be demonstrated in the same way. Fluids contained in vessels and pipes are in equilibrium when their surfaces are at an equal height. This is demonstrated as follows. Let us suppose **{Fig. 72}** that in vessel AT the surface area AB is ten times larger than that of pipe CD, and that AB descends to ab. It is clear that surface area CD of the connecting pipe will rise to cd,

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mechanics that is not the intended meaning here. These terms and concepts had yet to take on precise meanings.

and that the greater the surface area of the vessel compared to that of the pipe, the greater the height. Now if these two quantities of water are to be in equilibrium, the products of their masses and initial speeds must be equal. Now since the initial speed of the water in the tube is 10, whereas that of the vessel is 1, the mass in the tube must also be 10 times smaller. Consequently the heights of the fluids will be equal, since surface CD is only one tenth of surface AB.

565. In this way the relation between all kinds of power – which keep themselves in equilibrium by means of their initial speeds -- can be determined. All Statics, whether of fluids or solids, falls under this rule.

All Mathematicians agree on this principle. They always measure the relation among endeavors or dead forces by the product of masses and initial speeds, and no one has ever thought of calling this truth into doubt. But this is not the same as living force, that is, the force that resides in a body in actual motion, and which has a finite speed, that is a speed infinitely greater than that initial speed of which I have just been speaking.

566. Without revisiting the discussion of the measure of this living force, it is easy to see that this force is of a different kind[3] than dead force; that it must be infinitely larger than its element, and that it must be as a line is to a point, or as a surface is to a line.

{M. Leibniz is the inventor of living forces. *Acta Erud.* 1686 ff.} M. Leibniz was the first to discover the true measure of living force. **{One must carefully distinguish living force from its element.}** He very carefully distinguished these two forces, and he explained their differences so well that it became impossible to misunderstand the issue and confuse the two, if instead of rebelling against the discovery one had examined it.

567. We have seen (§560) that a pressure impresses on the body that yields to it an initial speed and an infinitely small force, and that this infinitely small force passes into the body on which the pressing cause acts.

This pressure is followed by another, and then yet another, and so on until the body -- having successively received an infinity of pressures all of which have an effect, and all of which the body conserves -- moves with a finite speed, and has acquired a force that is the sum of all the pressures accumulated and assembled within it.

{Fig. 73} Now no one can deny that for three equally strong and equally stretched springs AB, CD, EF, each has the same force, and that I can put one in place of the other without altering the effect that must result from the force of these springs. Thus, if one body has acquired all the force that resides in spring AB, and another body has acquired all the force that resides in the two other equal springs CD and EF, this second body will have twice as much force as the first; and a body that had the force of three of these equal springs would have three times more force

[3] "kind" here translates "genre", so Du Châtelet may be making a distinction at the level of genus rather than species.

than one that had the force of only one of these springs, and so on. Nothing seems more evident than this proposition, and if one wished to deny it I no longer know what there would be that is sure in human knowledge, nor upon what principle one could build in Philosophy; one might as well, it seems to me, renounce all enquiry.

{The living forces of bodies are as the square of their speeds. Proofs of this truth through the fall of bodies.} Gravity presses uniformly on gravitational bodies[4] at every instant and at all the points on their path during their fall towards the earth. **{Fig. 74}** I can therefore consider gravity, as regards its effects, as an infinite spring NR that presses on a body A equally throughout the space AB, and that follows it, always pressing equally on it and continually accelerating its motion towards B by the new pressures that it impresses on it at all the points that there are between A and B.

Now if we express the pressure that the body undergoes at A by the line A*m*; that which it receives at the closest moment *a* by the line *an*; and the next pressure by *bp*; and so on as far as B, where the body now is, we see that all these lines A*m*, *an*, *bp* etc. form the rectangle A*b*, and that the living force acquired at B must be represented by this rectangle, since it is composed of the sum of all pressures received during the time AB, pressures that lines A*m*, *an*, *bp*, B*b* represent. Thus, the living force of body A, having arrived at point B, will be to that of a body R descending from A to R as the rectangle A*b* is to the rectangle AL; that is to say, as the spaces AB to AR, since rectangles that have the same height are to one another as their bases (Euclid Book 6, Prop. 1).

The forces that the bodies have received at A and at R must necessarily be as the lines AB to AR since, as we have just seen, living forces must be to one another as the numbers of equal springs that are released and that have communicated their forces to the bodies in motion. Now, the number of these springs is evidently here as the spaces AB to AR since in double the space there is double the number of springs. Thus the living forces of the bodies that gravity makes descend must be to one another as the spaces AB to AR.

As we saw in chapter 13, Galileo's theory demonstrates that the spaces that gravity makes bodies traverse as they fall towards the earth are as the squares of the speeds. Therefore the living forces that bodies acquire in falling are also as the squares of their speeds, since these forces are as the spaces.

{How much this discovery was opposed initially.} This assertion seemed at first to be a kind of Physics Heresy. It was said, *Where would this square come from?* But one sees from what we have just said in the preceding sections that it is easy to deduce this from the accumulation of all the pressures that have acted on a body in a finite[5] time.

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[4] The French reads "les corps graves", indicating those bodies that behave gravitationally. See her chapters on gravity.

[5] This is printed as "infinite" in the 1740 edition, but it is obviously a mistake and was corrected in the second edition.

568. {All experiments have confirmed this.} Since this discovery (which we owe to Mr. Leibniz), all experiments have confirmed and have shown that in every case the force of bodies in actual and completed motion is proportional to the squares of their speeds multiplied by their masses. This estimation of forces has become one of the most fecund principles in Mechanics.

Philosophers are in agreement on the experiments that prove this estimation of living forces, and they all agree that the matter displaced, the springs compressed, the fibers flattened, the forces communicated, etc. – in short, all the effects of bodies in motion – are always as the square of their speed multiplied by their mass.

It would seem at first that there should be no dispute on this matter. For, since by common consent all force is equal to its fully executed effect, and since undisputed experiments prove that all the effects of bodies in motion are as the squares of their speeds multiplied by their masses, it seems unavoidable to conclude that the forces of these bodies are also as the squares of their speeds.

569. {Objection to living forces derived from the consideration of time.} The adversaries of living forces have thought they could evade this conclusion through the consideration of time which, they say, must always be the common measure of two forces that one compares. Now bodies that with double the speed make quadruple the effect do this in double the time. Therefore, they conclude, their force in equal time is only double, that is to say, proportional to the speed and not to its square.

{Response to this objection.} It seems to me that there is a very simple response to this Objection. For the ability to produce more effects and to act over more time is what I call, and what I believe everyone must call, *having more force*; and the total measure of this force must be what the body can do from the time it starts to move until the time it has exhausted all its force, no matter how much time this takes: the time must no more enter into this consideration than into the measure of the wealth of a man, which must always have been the same whether he spent it in a day, or in a year, or in a hundred years.

570. The question of the force of bodies must not turn on a metaphysical force that has neither employment nor resistance, for I do not know what the force is of a body that does not strive; if therefore nothing resists the force of a body, if it moves only with its mass and its speed, I know it only as *having speed*, and I cannot discover what force it has, nor what that force is.

But if this body encounters other bodies that it makes move, springs that it stretches, masses that it transports, displaces, or compresses, then I know it as *having force*, and I can estimate its force by the quantity of effects it produces in consuming this force, and cannot fear error in estimating this force through the effects that have consumed it.

{In which circumstances time is considered.} Time is to be considered on those occasions when during a longer period a greater effect can be produced, as in uniform motion. For the total space traversed (which is the only effect produced) will be greater or lesser depending on whether the motion of the body continues for more or less time. But a body that had the force to contract a given number of springs or to reascend to a given height, will never contract a larger number of similar springs and will never reascend to a greater height, no matter how long the time.

{Perpetual mechanical motion would be possible if in a longer time the same force could produce greater effects.} If with a longer time the body could produce a greater effect, as for example in reascending to a greater height than that from which it fell, the effect would be greater than its cause and perpetual mechanical motion would be possible, for it would only be a question of employing a sufficiently long time. But everyone considers perpetual mechanical motion to be impossible. Therefore when the force of a body is to be estimated, only the obstacles it overcomes count.

571. Thus, the force that is destroyed is always equal to the effect that it produces, no matter how long the force takes to produce the effect. For if this time is shorter and the resistance equal, the body consumes more force (and as a consequence, a greater part of this resistance) at every instant; and if the time is longer, the opposite happens. But in either case the same force is expended and the same quantity of effects produced, such that to overcome a resistance of magnitude 100, a hundred degrees of force are always required, no matter how much time is taken.

[572-86: see Zinsser]

587. Since the force of bodies in motion is proportional to their mass and to the square of their speeds, it follows that increasing the speed and mass of a body equally, increases its force unequally.

{The difference between our machines of war and those of the Ancients.} The Ancients made machines to break down walls whose mass was immense, and which with a very small speed produced a very large effect. Ours work in quite the opposite way, for the powder produces a very large effect by increasing the speed of a very small mass; and one reason for the superiority of our machines over those of the Ancients is that, the force of bodies increasing in proportion to the square of the speed but only in the direct proportion to the mass, this kind of increase produces a greater effect.

588. We have seen in this chapter that all experiments contribute to proving living forces, but Metaphysics speaks almost as strongly as Physics in their favor.

{Why Descartes gave false Laws of motion.} Descartes, in giving false Laws of motion, was misled in following a very fine principle, that of the conservation of an equal quantity of force in the Universe. This great Philosopher thought that Seneca's *semper jussit, semper paret*[6] better suited the Power and Wisdom of the Creator than being obliged to unceasingly renew the motion that he had once impressed upon his Work, as Mr. Newton thought.

This idea that seems so fine, so plausible, so worthy of the greatness of the Wisdom of the Author of Nature, cannot however be sustained when we make the force of bodies equal to their quantity of motion. For, independently of the case I mentioned in §586, following Mr. Newton, in which he brought about a continual production and annihilation of motion solely by change of position, Messrs. Huygens, Wren and others have demonstrated that motion can be increased or decreased to infinity when bodies collide, by placing the bodies that collide in a certain way, and by giving them particular masses.

But Mr. Leibniz, through his new estimation of forces, grants the Metaphysical principle Descartes discovered but did not apply well, and the Physical effects discovered more recently. For by distinguishing, as Leibniz did, between the quantity of motion and the quantity of the force of bodies in motion, and by making this force proportional to the product of mass times the square of the speed, we find that although the motion in the Universe varies at each instant, the same quantity of living force is nevertheless always conserved in it. For force is not destroyed without an effect that destroys it, and this effect can only be the same degree of force communicated to another body, since the body that takes always removes from the other body as much force as it itself receives. Thus, the production of the smallest degree of force in a body is necessarily accompanied by the loss of an equal degree of force in another body, and vice versa. Thus, force cannot perish, either wholly or in part, without reappearing in the effect that it produces, and from this all the Laws of motion can be drawn.

{The equal conservation of living forces is one very strong reason in their favor.} Now this conservation of forces would be a very strong Metaphysical reason, all things being equal, to determine and estimate the force of bodies in motion by the square of their speeds. For when we seek the effects of force we do not find the product of mass by speed, but the product of mass by the square of its speed. Now, though motion perish and be reborn, there is nothing in this that is contrary to good principles provided that the force producing this motion stays the same. For you saw in Chapter 8 that speed is a mode of motive force: now when speed increases or decreases nothing substantial is created or annihilated, the body's motive force is merely modified by the variation of the speed, and this force itself, that is something real that endures as matter, can be neither destroyed nor produced anew. For it is easy to show geometrically that in everything that happens between elastic bodies, however they collide, the same quantity of force is unalterably conserved, if we take for the force the product of the square of the speed by the mass. But were the forces of moving bodies not in this ratio, living forces – which are the source of motion in the Universe – would not be conserved.

[6] Du Châtelet: He has commanded once, and he obeys always that which he has ordained.

589. {The work[7] of force in the collision of elastic bodies.} It is true that it is only in elastic bodies that the force of bodies in motion can be tracked and calculated in its entirety, because after the collision these bodies return to the same state they were in beforehand, and one can find the work of their forces in the other bodies that they have put into motion, or whose motion they have increased without altering their shape.

590. {And in the collision of bodies that have no elasticity.} As regards what happens between bodies that cannot return to their former state, this is one of those cases where it is not easy to track living force because it is consumed in displacing the parts of the bodies, in overcoming their cohesion, in breaking up their structure, stretching perhaps the elasticity among their parts, and who knows what else? But what is very certain is that force does not perish; it can in truth appear lost, but we would always find it again in the effects it produces if we were able always to perceive those effects.

[7] As noted above, we translate "effort" and "efforts" as "work", noting that "work" has a technical meaning in mechanics that is not the intended meaning here. These terms and concepts had yet to take on precise meanings.