

Émilie Du Châtelet, *Foundations of Physics*, 1740.

Translated by Katherine Brading *et al.*¹ at the University of Notre Dame and Duke University. Footnotes are ours except where otherwise indicated.

Du Châtelet's marginal notes are placed in **{bold}** in the closest appropriate place in the text. Please see the French original for the position of each note in the margin alongside the paragraph. Figures are available in the original text, and online via the BNF.

Chapter 13. Of Heaviness²

293. {Definition of Heaviness.} We call Heaviness the force by which every Body being left to itself, falls toward the surface of the earth.

294. This same force, that makes bodies fall when they are not supported by anything, makes them press against the obstacles that resist them and prevent them from falling: thus, a stone has weight on the hand that supports it, and falls along a line perpendicular to the horizon should that hand come to abandon it.

295. {Gravity produces a dead force or a living force, according to the circumstances in which it acts.} The force that brings Bodies to fall therefore engenders in the Bodies a force that is either a **dead force** or a **living force**, according to the circumstances in which it acts.

296. When Bodies are resisted by an **invincible** obstacle, the gravity that makes them press against this obstacle in this case produces a dead force; for it does not produce any effect.

297. But when nothing resists the Bodies, gravity in this case produces a living force in these Bodies, since it makes them fall toward the surface of the earth.

298. It has throughout time been perceived that certain Bodies fall toward the earth when nothing supports them, and that they press the hand that prevents them from falling; but as there are some whose weight seems imperceptible, and that rise, whether on the surface of water or on that of air, such as feathers, very light wood, flames, exhalations, etc., whereas others sink, such as stones, earth, metals, etc., Aristotle, the father of philosophy and of error, imagined two appetites in Bodies. **{Aristotle's opinion on heaviness.}** Heavy bodies had, according to him, an appetite for arriving at the center of the earth (which he thought was the center of the Universe), and that light bodies had a completely contrary appetite that moved them away from this center, and that carried them upward on high.

¹ Especially Anne Seul, Jeremy Steeger and Penelope Brading.

² We translate "pesanteur" as heaviness and "gravité" as gravity.

But it was soon recognized how chimerical were these appetites of Bodies; and positive lightness was one of Aristotle's mistakes, of which we have utterly disabused ourselves.

299. {Heaviness belongs to all Bodies.} Heaviness being recognized as pertaining to all perceptible Bodies, and positive lightness being banished, was already a great deal, since this was one error less; but there remained many truths to discover about this property of Bodies, and about its effects.

300. Aristotle, that is to say, everyone (for before Galileo we knew hardly any proof of truth other than the authority of Aristotle), Aristotle, I say, thought that different Bodies fell in the same medium with speeds proportional to their mass; but Galileo challenged this error, and dared to assert, despite Aristotle's authority, that the resistances of the media in which Bodies fall was the sole cause of the differences found in the times of their fall toward the earth; and that in a medium that does not resist at all, all Bodies, no matter their nature, would fall equally fast: *Che se si levasse totalmente la resistenza del mezzo, tutte le materie descenderebbero con eguali velocita.*³

301. {Experiment that made Galileo think that all Bodies would fall in the same time without the resistance of the medium.} The differences that Galileo found in the times of fall of several moving Bodies that he let fall through the air from a height of 100 cubits [coudées], brought him to this assertion, because he found that the differences were too insignificant to be attributed to the different weights of the Bodies.

Having in addition let fall the same moveable Bodies in water and in air, he found that the differences in their respective falls in the different media responded, more or less, to the densities of these media, and not to the mass of the Bodies; therefore, Galileo concluded, the resistance of the media, and the size and roughness⁴ of the surface of different Bodies, are the only causes that make the fall of some faster than that of others.

{Lucretius guessed this truth.} Lucretius himself, however bad a Physicist he was otherwise, glimpsed this truth, and expressed it in the second Book by these two lines.

*Omnia qua propter debent per inane quietum
Aequae ponderibus non aequis concita ferri.*⁵

302. {Experiment that made Galileo suspect that Bodies, as they fall toward the earth have an accelerated motion.} One discovered truth almost always leads to another: Galileo, having

³ *That if one completely removed the resistance of the medium, all matter would descend with equal velocity.*

⁴ The word in the French text is "scabrosité", which we have been unable to find in a French dictionary.

⁵ *De Rerum Natura* II, 238-9: *wherefore all things must needs be borne on through the calm void, moving at equal rate with unequal weights.* Translation *Lucretius on the Nature of Things*, Cyril Bailey, OUP, 1910.

noticed furthermore that the speeds of the same moveable Bodies were greater in the same medium when they fell from a greater height, concluded from this that since (the weight of the Body and the density of the medium remaining the same) the different height brought changes in the speeds acquired in falling, the Bodies must naturally have an accelerated motion toward the centre of the earth: Here is how he expresses himself in the first Dialogue {p. 56}: *Dico per tanto che un corpo grave ha de natura intrinseco principio di muoversi verso 'l comun centro de i gravi eioe del nostro globo terrestre, con movimento continuamente accelerato.*

It was this observation that led Galileo to research the Laws that would be followed by a body falling toward the earth with a uniformly⁶ accelerated motion.

303. He therefore supposed that the cause (whatever it may be) that makes heaviness, acts equally at each indivisible instant, and that it impresses upon the bodies that it makes fall toward the earth a motion equally accelerated in equal time, such that the speeds they acquire in falling are as the times of their fall.

It is from this single supposition that is so simple and conforms so well to nature's genius⁷ that this great Philosopher has extracted the entire theory of falling bodies of which I will give an account, the Theory that is now adopted by all Philosophers, and of which every experiment has become a demonstration.

304. {Demonstrations that originate from this supposition.} The Space traversed in one second by a body falling toward the earth by the force of gravity can be represented by the area of triangle ABC {Plate 5, Fig. 28}, as I will demonstrate as follows. Suppose then that this Space ABC is traversed by body A in an equally accelerated motion, in the time represented by line AB, which time I have supposed to be one second, and that line BC represents the sum of the speeds acquired at the end of this second. If the force, whatever it may be, that accelerates the body toward the earth ceased to act when the body arrived at point B, it is certain that this body, by the force of inertia, would continue to move in a uniform motion with the speed BC acquired at point B (Second Law, §229). Now in uniform motion, the Space traversed is the product of the speed and the time (§241). Therefore the space that the moveable body A would traverse with uniform motion during the same time of one second, and with the speed BC, would be the parallelogram BCDE formed by the line BD=AB that represents the time, and by the line BC that represents the speed; but this parallelogram is double the triangle ABC that I supposed was traversed by the body with an accelerated motion in the same time AB, for this triangle and this parallelogram have the same base and the same height (Euclid, Book One, Prop. 41). Therefore, if the accelerative cause should cease, the space that the body would traverse with uniform motion (with the sum of the speeds acquired by the acceleration) would be, in equal time, double

⁶ Here, Du Châtelet uses “equally”, but the standard terminology is “uniformly”, which she uses elsewhere.

⁷ By “genius” here Du Châtelet might mean “ingenuity”, but it is more likely that she means nature's character or spirit.

the space that this body would have traversed with an accelerated motion in acquiring this same speed.

305. Therefore, in the second instant, solely through the speed acquired at point B and independently of the actual effect of its heaviness, body A will traverse the space BCDE {**Fig. 29**}, double the space ABC traversed in the first instant; but the cause that makes these bodies fall being supposed to act equally at each indivisible instant, this body in the second instant⁸ will acquire a second degree of speed equal to that which made it traverse space ABC in the first instant; it will therefore traverse in the second instant a space three times the space traversed in the first; that is to say, the space BCDE (double the space of ABC) by a uniform motion, and the space CEF=ABC by the acceleration impressed by gravity in the second instant.

306. This body, for the same reason, will traverse in the third instant a space five times the first, and a space seven times in the fourth, and so on; and consequently the spaces that this body will traverse in falling for equal and consecutive times 1, 2, 3, 4, etc. will be as the odd numbers 1, 3, 5, 7, etc., and this is easy to see simply by inspection of Figure 29.

307. But these odd numbers, whose progression represents the unequal spaces traversed by the moveable body in a uniformly accelerated motion in equal time, these numbers being added to each other at the end of each of these times, form the natural sequence of the squared numbers 1, 4, 9, 16, of which the numbers 1, 2, 3, 4, that represent the times and speeds, are found to be the roots; for $1 \times 1 = 1$, $2 \times 2 = 4$, $3 \times 3 = 9$, $4 \times 4 = 16$, etc. The spaces that bodies traverse in falling toward the earth must therefore be as the square of the times of their fall, and of the speeds acquired in falling, if they fall in a uniformly accelerated motion, as Galileo had supposed.

One must always find the same proportion between the space and the time, from the first moment of the fall until the last, for any time whatever. Thus, the body at the end of the fifth instant, for example, will have traversed a space of 25, at the end of the seventh a space of 49, and so on.

308. As regards what I supposed (§304), that the space traversed by body A in an accelerated motion during the first instant could be represented by the area of the triangle ABC, it is easy to show the truth of this.

{**Plate 5, Fig. 30.**} For we showed you, in Geometry, that when one erects on a right line AB several other right lines, such as DE, BC, such that AD is to DE as AB is to BC, the ends C and E of these lines are in a same right line AC, and that the Figure is a triangle, because the triangle is the only figure having this property of proportional sides.

Now, we have seen (§303) that in Galileo's theory, the times are as the speeds, that is to say, that the time needed by the moveable body to acquire a given speed is to the time needed by it to acquire another speed as the first speed is to the second; thus, in expressing the time of the falls

⁸ Each "instant" here is a one second time interval. Du Châtelet uses "instant" at the beginning of this paragraph, and then "second", but we translate "the second second" as "the second instant".

by the lines AD, DB, we must represent the respective speeds acquired during these times by the lines DE, BC, from which the triangle ABC will result by the proposition of Geometry that I just cited. Now, this triangle ABC represents the space traversed by the moveable body in its fall during time AB, for you have seen in chapter 11 (§241) that in uniform motion the space traversed is the product of the speed and the time; you have also seen in the same chapter (§242) that in an infinitely small instant, motion is always uniform. Therefore the space traversed in the first infinitely small instant will be an infinitely small parallelogram formed by the line that represents time and by the line that represents speed. Now the entire triangle ABC can be considered as being divided into infinitely small parallelograms, the sum of which will form the triangle ABC, according to the cited proposition. Thus the area of this triangle can represent the space traversed by the moveable body in a given finite time of its fall, as I supposed in §304.

309. It is very possible that bodies, when falling, traverse a very small space without accelerating their motion, for the reason that time is needed to produce all natural effects; but if this is the case, it is impossible for us to perceive it because of the extreme smallness of this space; thus, this changes nothing in the demonstrations above.

310. {Experiment carried out by Galileo, in which he found that bodies, when falling toward the earth by their own heaviness, traverse spaces that are among themselves as the squares of the times.} Galileo, having demonstrated what must happen to a moveable body that would fall toward the earth in an equally accelerated motion, sought to ascertain through experimentation that nature really follows this proportion in the fall of weights. To achieve this, he devised a very ingenious experiment. He made a large wooden tube twelve cubits long and about an inch wide; to the inside of which he glued a very light parchment, so that it was as smooth as it could be; and having raised the upper end of this channel on a horizontal plane to the height of one, two, and successively of several cubits, in such a way that this channel became an inclined plane, he let a small copper ball, perfectly round and perfectly polished, fall along the channel, and in making it fall successively the entire length of this channel, or a quarter of it, or a half, he always found in his experiments, that he asserts he had repeated up to a hundred times, that the times of the fall were as the square root of the spaces traversed. Now, in making an inclined plane of this channel along which the ball was falling, Galileo slowed the motion of the moveable body, and by this means made the speed discernible, which would not have been possible in a perpendicular fall that short; for bodies fall more slowly along an inclined plane than along a perpendicular plane, and they follow the same laws in both of these falls (§425 & §428). Thus it was easy for him to know by this means what space heaviness made a moveable body traverse during a certain time, and he measured this time by the quantity of water that flowed from a vessel while the body traversed these different spaces.

311. {Experiment of Riccioli and Grimaldo, that confirms that of Galileo.} Riccioli and Grimaldo, sought, as had Galileo, to ascertain this truth by experiment. They made moveable

bodies fall from the top of several towers of different heights, and they measured the time of the fall of these bodies from these different heights by the vibrations⁹ of a pendulum, the accuracy of which Grimaldo ascertained by counting the number of its vibrations from one passage of the tail of the Lion through the Meridian until the next.¹⁰

These two Jesuit scholars found from the result of their experiments that these different heights were exactly as the squares of the times of the falls.

312. {The oscillations of the pendulums confirm this discovery.} The times of the oscillations of the pendulums that are always the square root of their different lengths, are another demonstration of this truth; for heaviness is the sole cause of these oscillations.

313. {The truth of this discovery of Galileo is unanimously recognized.} Thus, this discovery of Galileo became, through experiments, the fact of Physics of which we are the most sure; and all Philosophers, despite the diversity of their opinions on almost everything else, agree today that bodies in falling toward the earth traverse spaces that are as the squares of the times of their fall, or as the squares of the speeds acquired in falling.

314. {Machine of Father Sebastien, that shows to the eye this discovery of Galileo.} Father Sebastien, that Geometrician of the senses, devised a Machine made of four equal parabolas that intersected at their tips; and by means of this Machine, of which we find the description and diagram in the *Mémoires de l'Académie des Sciences* A. 1699, he demonstrated to the corporeal eye, the witness of which the mind's eye almost always needs, that the fall of bodies toward the earth operates according to the progression discovered by Galileo.

315. {Truths that arise from the discovery of Galileo.} It is therefore most certain, following this discovery:

1. That the force that makes bodies fall is always uniform, and that it acts equally upon them at each instant.
2. That bodies fall toward the earth in a uniformly accelerated motion.
3. That their speeds are as the time of their motion.
4. That the spaces that they traverse are as the squares of the times or as the squares of the speeds; and that consequently the speeds and the times are as the square roots of the spaces.
5. That the space that the body traverses in falling during a given time is the square root of that which it would traverse during the same time with a uniform motion that is the sum of the

⁹ That is to say, oscillations. Du Châtelet uses the term “vibrations” here, and then “oscillations” in the next section.

¹⁰ The “Lion” is the constellation Leo, and the pendulum accuracy is being ascertained with respect to the sidereal day (as measured by the time interval between the tip of Leo's tail passing through the Meridian from one day to the next).

acquired speeds; and that consequently this space is equal to that which the body would traverse with a uniform motion that is half of these speeds, etc.

6. **{Gravity is that which gives weight to bodies.}** That the force that makes bodies fall toward the earth is the sole cause of their weight; for since it acts at each instant, it must act on the bodies whether they are at rest or in motion; and it is through the unceasing efforts that bodies make to obey this force that they weigh upon the obstacles that restrain them.

316. {It acts equally upon bodies in motion and upon bodies at rest.} Gravity acts equally on bodies at each instant, whether they are at rest or in motion; and the speed that it impresses upon them is equal in equal time, regardless of the speed that they have already acquired (§315, num. 3).

317. {Bodies begin to fall with an infinitely small speed.} Since gravity acts equally at each instant upon bodies, whether they are at rest or in motion, bodies begin to fall with an infinitely small speed, with which they tend to fall toward the earth, before the obstacle that restrains them is removed. Thus, Mr. Mariotte was mistaken in the eleventh Proposition of the second Part of his Treatise on Percussion, when he concluded from an experiment that he reported there, that the speed with which bodies begin to fall is not infinitely small. For if this speed was not incomparably smaller than any finite speed, the speed of a body that falls would have to be infinitely great in a finite time; but a body in falling does not acquire an infinite speed in a finite time: therefore, etc.

318. If the direction of a body that fell from any given height happened to be changed without its speed being altered, such that this body, instead of continuing to descend, happened to reascend, it would reascend with a uniformly retarded motion; for this body having fallen from A to E in two seconds, for example **{Fig. 31.}**, must conserve by its force of inertia the speed acquired at E, unless some cause happens to remove the speed from it. Now by this speed acquired at E, the body would traverse in a uniform motion in two seconds the space ED, double the space AE traversed in an accelerated motion in falling. But since gravity acts equally on bodies, whether they are at rest or in motion, whether they are ascending or descending (§315, num. 1), this body will have in reascending a motion composed of the uniform motion that it would have had independently of the actual action of gravity, and of the motion that gravity impresses upon it at each instant. But this motion impressed by gravity, that would accelerate the motion of this body when it was descending, must slow it down when it ascends, since the action of gravity is always directed down toward the earth, away from which this body distances itself in ascending.

{Bodies in falling from any given height acquire the force necessary to reascend to the same height.} This body will therefore have in reascending a motion equally retarded in equal time. Thus, in the first instant,¹¹ were the body to move uniformly with the speed acquired at E, it

¹¹ Once again, we use “instant” for the time interval of a second, in order to avoid “the first second” and “the second second”.

would in reascending traverse AE. However, the body reaches only as far as C, for as it reascends gravity removes all the speed that it had given to it in the first instant of its fall. Likewise, once this body arrived at C, if gravity ceased to act upon it and to pull it back downward, it would in reascending traverse in the second instant the space CF, double the space AC, for the speed that made it traverse the space AC in descending is all that then remains. But since gravity acts always equally, this body will arrive only at A in this second instant: gravity will diminish the body's speed in the same proportion that it had increased it in falling; and consequently the total space that this body will traverse in reascending during the two instants will be equal to that which it had traversed in descending.

319. It follows from this:

1. That a body in falling acquires by the action of gravity speeds capable of making it reascend in equal time, despite the unceasing efforts of gravity to pull it back down, to the same height from where it fell, assuming that something changes its direction without altering its speed: this is what is seen in the oscillations of pendulums (§445).

2. That the body in reascending will traverse spaces in inverse ratio to those that it traversed in descending; such that since the spaces traversed in descending during the times 1, 2, 3, etc., are 1, 3, 5, etc., the spaces traversed in reascending during the same times will be 5, 3, and 1. For in the first case, the speed of the body increases in each instant, whereas in the second, the speed diminishes in each instant; thus, gravity retards the motion of reascending bodies in the same proportion as it accelerates that of falling bodies.

And finally 3. That a body that is thrown upwards ascends until gravity has made it lose all the motion that had been impressed upon it for ascending; and that consequently this body will reascend to the same height at which, had it fallen from this height, it would have acquired in falling by the force of gravity a speed equal to that which was imparted to it for reascending.

320. Thus, the heights to which bodies can reascend by the speed acquired in falling are always as the square of their speeds; and two bodies that reascend with unequal speeds would reascend to heights that would be between them as the squares of these same speeds.