Philosophy, Science, and History

A Guide and Reader

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CHAPTER 21

On the Divisibility and Subtlety of Matter

ÉMILIE DU CHÂTELET

Translation for this volume by Lydia Patton[1] of pages 179–200 (Chapter 9) of *Foundations of Physics.* Paris: Chez Prault Fils. Original publication date 1750, original title *Institutions de Physique.* Most remaining chapters of *Foundations of Physics* are translated in Du Châtelet 2009/1750.

§165 Extension can be conceived in terms of length, width, and depth; thus, the line AB is extended in length, the surface ABDE is extended in length and in width, and the cube ABCDEFGH is extended in length, width, and depth: these are the three dimensions of extension. (See Figures 7 and 10 below.)

§166 Every body has these three dimensions, or to speak with exactness, there are nothing but solids in nature. But since our mind[2] has the power to make abstractions, we can consider length without taking note of width nor depth, similarly, we can consider length and width only, without taking note of depth, and geometry is founded on these abstractions of our mind. Surfaces, lines, and points thus are not matter, but one conceives them in matter by abstraction.[3]

§167 However, to assist the imagination, and to form for oneself a distinct idea of the three dimensions of extension, one can imagine two points A and B at a distance from each other, and one can imagine that if the point A, in travelling to meet the point B, leaves a production of itself in each part of the interval that separates the points, it will form the line AB, which one supposes to be extended only in length.

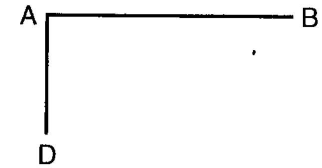

Figure 21.7 (redrawn)

One can suppose further that the line AB, flowing[4] the length of the line AD, leaves a production of itself in the entire path that it traverses to travel from the point A to the point D, it will form the surface ABDE, which one supposes to be extended in length and in width.

Finally, if the surface ABCDE flows along the surface CDEF they will form the cube ABCDEFGH which has the three dimensions of nature, since it is extended in length, width, and depth.

§168 Most philosophers,[5] having confused the abstractions of our mind with physical body, have wanted to demonstrate the divisibility of matter to infinity by means of the reasoning of the geometers on the divisibility of lines that one pushes to infinity. This has given rise to the famous labyrinth of the divisibility of the continuum that has so embarrassed the philosophers. But they could have avoided all the difficulties that this divisibility involves if they had taken care never to apply the reasoning that one applies to the divisibility of the geometrical body to natural and physical bodies.

§169 The geometrical body is nothing but simple extension. The geometrical body has no determinate and actual parts, it contains nothing but simply possible parts, which one can increase to infinity as one wills; because the notion of extension contains nothing but parts coexisting and united, and the number of these parts is absolutely indeterminate, and does not enter into the notion of extension. Thus, one can determine this number as one wills, without harming the extension. That is to say that one

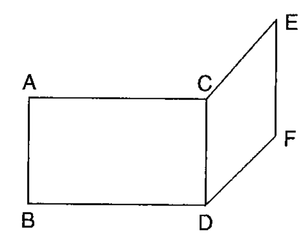

Figure 21.10 (redrawn)

can establish that an extension contains ten thousand, or a million, or ten million parts, and so on, depending on how one wishes to define some part or another as *one*. Thus, a line contains two parts, if one takes its half to be *one*, and it has either ten, or a thousand, if one takes the tenth or thousandth part as the unit. Hence, this unit is absolutely indeterminate, and depends on the will of whomever considers this extension.

§170 Each abstract and geometrical extension can thus be expressed by some number or other, but things are quite different in nature. All that exists naturally must be determined in every way, and it is not in our power to determine it otherwise.[6] A watch has its parts, for example, but these are not parts that are determinable merely by the imagination, these are real, actually existing parts. It is not open to say, *This watch has ten, one hundred, or a million parts*; for, as a watch, it has one number that constitutes its essence, and it cannot have more or fewer parts and remain a watch. It is the same with all the natural bodies, they are all machines which have determinate and dissimilar parts, which one is not permitted to express with just any number.

§171 It is in confusing geometrical extension with physical extension, and in supposing that physical extension is composed of an infinity of extended parts, that the Ancients formed those false and specious arguments against the possibility of movement.

The most famous of all the paradoxes[7] was the one that Zeno had called the *Achilles*, in recognition of Achilles' invincible force. He proposed Achilles running toward a tortoise, and, as Achilles ran ten times faster than the tortoise, he gave the tortoise a head start of one league, and he reasoned as follows: while Achilles traveled one tenth of a league, the tortoise would travel one hundredth of the league; thus, from tenth to tenth, the tortoise would always remain in front of Achilles, who would never catch up with it.[8]

First, if it were true that Achilles would never catch the tortoise, it would not follow from that that movement would be impossible. Since Achilles and the tortoise really move, Achilles always approaches the tortoise, who is assumed to always be ahead of him, albeit infinitely little.

But, secondly, since this ingenious paradox is founded on the divisibility of extension to infinity, the principle of sufficient reason provides a sufficient refutation. For it is proven by means of this principle that physical extension ultimately is composed of simple beings, and that consequently its divisions, even the possible ones, have positive and real limits.[9]

Entire treatises have been written to resolve Zeno's paradox. To refute it, perhaps it suffices to walk in his presence as Diogenes did; but failing such a de facto response, one can see whether it is easy to find one de jure.

Grégoire de Saint Vincent was the first to demonstrate its falsity, and who assigned the precise point at which Achilles would have to reach the tortoise, and this point was found, by means of infinite geometrical progressions, to be one and one ninth leagues. The sum of any infinite geometrical progression is finite, and this is because infinite being, and infinite extension, are two very different things. For any finitude whatsoever, a foot, for example, is a composite of finite and infinite: the foot is finite, insofar as it contains nothing but a certain number of simple beings, but I can suppose it to be divided into an infinity, or rather into a non-finite number[10] of parts, considering this foot as an abstract extension. Thus, if I were to consider half of the foot first, then half of what remains or a quarter of a foot, then half of that or an eighth of a foot, I would proceed thus, mentally, to infinity; always taking new and decreasing halves. These, all together, would never make up that foot, which would then become a geometrical body, because of all its properties I would only retain in my mind that of the extension on which my ideal division is performed.

Thus, the divisibility of extension to infinity is at the same time a geometrical truth and a physical error. And, thus, all the reasoning on the divisibility of matter drawn from the nature of asymptotes, from the incommensurability of the diagonal of a square, from infinite series, and from other geometrical considerations, are absolutely inapplicable to natural bodies, as are the theorems of M. Keill, by which he claims to prove that one could fill the entire universe with a grain of sand.[11] For one must not admit anything into physics besides actual parts, whose existence can be demonstrated by experience or by rigorous demonstration.

§172 One has seen above[12] that indivisible atoms, or parts, of matter are inadmissible, if one considers them as simple, irresolvable and primitive matters, because one cannot give a sufficient reason for their existence. But as long as one recognizes that they derive their origin from simple beings, one certainly can admit them. For it is very possible, and experience renders it very likely, that there is a certain determinate number of parts of matter in the universe which nature never resolves into their principle, which remain undivided in the present constitution of this universe, and that all the bodies that compose the universe result from the composition and the mixture of these solid particles, so that one can regard them as elements endowed with shapes and internal distinctions that result from their parts.

That nature halts in the analysis of matter at a certain fixed and determinate degree, is sufficiently probable, on the basis of the uniformity that rules nature's works, and on an infinity of experience.

1. If matter were resolvable[13] to infinity, it would be impossible that the same germs and the same seeds would consistently produce the same animals and the same plants, that plants and animals would always take exactly

the same time to grow, that they would always conserve the same properties, and that they would be the same at present that they were before. For if the substance that nourishes them were at times more subtle, and at times more coarse, it would be impossible that they would not be subject to perpetual variation. Because if the parts of the substance were more subtle, then it would take longer for the same body to develop than if the substance were more coarse, and consequently, the body would be more or less solid, and would develop in more or less time, depending on whether the parts of the substance that feeds it were more subtle or more coarse; and it follows that the form and the manner of being in composition would be subjected to a thousand changes, and the species of things would be blurred constantly.

But there is no such disturbance in the universe: plants, animals, fossils, each finally produces its likeness with its attributes, which constitute its essence: thus matter is not resolvable to infinity.[14]

2. If the parts of matter were resolvable to infinity, not only would species intermingle, but every day new ones would be formed; but no such new species is formed in nature, even monsters only produce their own. The hand of the creator has marked the limits of each being, and these limits will never be broached; however, if matter were divisible to infinity, they would be broached at each moment. The order that reigns in the universe, and the conservation of that order, thus appear to prove that there are solid particles in matter.

3. The dissolution of bodies has fixed limits just as their development does. The fire of a burning mirror, the most powerful solvent that we know, melts gold, pulverizes it, and then vitrifies it.[15] But the effects do not go beyond that, while, on the other hand, if matter were resolvable to infinity, the fire should destroy everything; and one could say neither why liquids only ever acquire a certain degree of heat, nor why the action of the fire on the body has such precise limits, if actual solidity and irresolvability were not bound up with the parts of matter, when they go beyond a certain littleness, and if those parts did not oppose an insurmountable barrier to this powerful agent with their solidity.

4. This irresolvability of the first bodies becomes indispensably necessary, if one adopts the system of germs, which the new discoveries that have been made with the microscope seem to demonstrate. All the world knows those discoveries of Hartsoëker, and every day it becomes more likely that nature does not act except through development.[16] Thus, if each grain of wheat contains the germ of all the grains of wheat it will produce, it is necessary that the actual divisions of matter have limits, even if those limits are unassignable by us.

It is thus strongly likely that there are particles of matter of a certain determinate littleness, which nature does not divide further.

§173 If one asks for the sufficient reason of this actual irresolvability of the little bodies of matter, it would be easy to find it in the mutual movements of its parts, for mutual movements are the cause of cohesion, according to Leibniz.

§174 Even though the actual divisions that matter can undergo have real limits, experience reveals to us a subtlety in the parts of natural bodies that stuns the imagination, and which one scarcely knows how to admire sufficiently. Wolff[17] has observed five hundred eggs within a grain of dust, from which are hatched animals similar to fish, and in which one finds an infinity of parts, as with the largest animals of the sea.

The same author reveals that a grain of barley can contain twenty-seven million living animals, each of which has twenty or twenty-four feet, and that the smallest grain of sand can be the habitat of two hundred ninety-four million organisms, which propagate their species, and who have nerves, veins, and fluids that compose them, and which no doubt have the same proportion to the bodies of these animals that the fluids of our bodies have to our mass.

The work of the drawers and beaters of gold provides fine proof of the sublety of the parts of matter. Boyle reports that a single grain[18] of gold beaten into gold leaf fills a space of fifty square inches measured geometrically, but if one divides a side of one inch into two hundred parts, or of a line[19] in twenty, which still results in parts visible to the eye without a microscope, each square inch will have forty thousand parts of gold that one still can distinguish without a microscope, and consequently the entire leaf will have two million parts visible to the unaided eye. If one adds to this, that such a leaf is still divisible in thickness into at least six leaves, as one can conclude from the observations of Réaumur, who observed that the thickness of a leaf of gold is about 1/30000th part of a line, and the thickness of silver wire is 1/175000th part of a line, consequently the silver wire is about six times less thick than gold leaf.[20] Thus, this gold leaf, reduced to the thickness of silver leaf, would be divided in six parts, from which it can be seen that each grain of gold contains about twelve million parts visible to the unaided eye. Thus, since these parts are nothing but gold, and remain gold when one observes them using microscopes that magnify an object twenty or thirty thousand times, and which consequently reveal thirty thousand more parts in each of these twelve million parts that the unaided eye can distinguish in this grain of gold, one can conceive the point of fineness to which nature subdivides matter. For gold is a mixture of other, finer matters that are not gold, and it contains an infinity of pores that are filled with another matter besides the matter proper to it. Since one no longer distinguishes the constitutive parts of gold after this enormous division, nor the matter which passes through its pores, one can hardly expect to see the shapes and the movements of those parts of the mixture that must contain the immediate reason for the qualities that we notice in gold, and those parts themselves are composed again of simple beings.

§175 These considerations show us that the subtlety of the parts of matter is inexpressible, and that there is no one who can ever determine the number of parts of which a grain of sand is composed, since this number goes beyond our imagination and all that we can represent to ourselves. As reason shows us that this division has no limits, and that matter never ceases to be divisible as long as it is matter, one can say that, as far as we are concerned, matter is not only divisible, but divisible to infinity, although in reality the divisions have limits. For these divisions are so remote that they stretch to infinity for us, because for us, the infinite is a quantity that no number can express.

§176 Thus, it is evident that in nature there is an infinity of differently configured and differently altered matters, which escape our senses and our observations by their littleness, and which nonetheless produce the phenomena that we observe; and the first reasons of physical qualities are all found in these differently configured matters that are impossible for us to distinguish. We must conclude from this that an infinity of events could take place in the smallest space, just as in the entire world, but human attention could never perceive them, and it is a great deal for our understanding even to have comprehended their possibility. So it is a waste of time to attempt to divine these imperceptible mysteries, and we should limit ourselves to observing carefully the qualities that fall under our senses and the phenomena that result, which we can employ to make sense of other phenomena that depend on them.

§177 All bodies contain two types of matter, proper matter and foreign matter. Proper matter can be constant or variable. Constant matter is that without which the body could not subsist. Variable matter, like air and water, for example, is that which sometimes comes to rest in the largest pores, and which augments the weight of the body in introducing itself into and resting between its parts. All the proper matter of a body rests, moves, is weighed, and acts with it; but foreign matter doesn't always move with the body, but rather passes freely among its pores, as water does through a box pierced with several holes.

§178 The reality of the existence of the two matters is easily demonstrated by experience; for experience teaches us that bodies have different densities and different weights. Water, for example, weighs more than air, and gold is denser than wood, and weighs more.

All matters, including even gold, the densest of all, have pores that are not filled with the same matter as their proper matter. And, there being no point of absolute vacuum in the universe, it is necessary that these pores be filled with foreign matter that is not weighed with these bodies, and which does not enter into collisions with them if they encounter other bodies in their path, but which fills all their interstices, and which moves among them with as much liberty as air through a screen, or water through a net.

§179 This can be proven again by considering cohesion. For since the principle of sufficient reason rules out a vacuum between the parts of bodies, and shows that there will not be two parts of matter, one indiscernible from the other, in the universe, hence there cannot be any shape or diversity in nature except by means of motion. If all the parts of matter rested upon each other, it is evident that only a perfect, similar continuity, without any shape, would result. Thus, it is necessary, not only that all matter be in motion, but that its motion be varied to infinity in its speed and in its direction; so that the different qualities and all the internal differences of the parts of matter can emerge from it. Thus, if several parts of matter seem to be without force and at perfect rest, it must be the case that the movement of these parts tends in opposite directions with the same force; and that consequently they come to rest in the same place, which is what makes for cohesion. For we know that two bodies, strongly pressing against each other, cannot be separated except with difficulty, and appear to make up only one body. Thus, mutual movement is the origin of cohesion, according to Leibniz and his followers. So we have seen that the degree of speed with which a body moves, and the direction of its movement, are not determined by anything but the movement of several other bodies that contain its sufficient reason.[21] In this way, after the parts move in opposite directions with equal speeds, and after they cohere in this manner, it is necessary that the movement of an external matter, which does not cohere with these parts, must determine their direction and their speed. There are matters, very fine and very rapidly moving, which hide from our senses, and which produce many of the effects that we notice. Magnetic matter and electric matter are likely in this category, as well as the matter of fire, of cohesion, of elasticity, of gravity, and doubtless of an infinitude of others that are differently modified, and which cooperate in diverse ways to produce the sensible qualities of bodies.

§180 These reflections should be a warning against the impulsiveness of various philosophers, who, when they see phenomena like the fluids proposed by some, which cannot be explained at present, cut the tangle that they should unravel instead, and decide that no such fluid can exist, nor can it produce the effects that we observe.[22] For to make such a decision, it is necessary to know all the ways that matter can be moved, and all that can result from all its diverse motions, but we are still far from this.

§181 The only experiments with electricity show well enough which singular effects nature can produce by the motion of subtle matter, although the way in which she employs them to produce the effects is inexplicable for us.[23] For these matters are not sensibly perceived during the experiments with electricity, nonetheless, those who undertake to explain all the phenomena of electricity mechanically, by means of motion and a very subtle fluid, undertake a problem infinitely more difficult than that of the cause of planetary motion. For in planetary motion, a great regularity and a great uniformity rule, but the phenomena of electricity vary almost to infinity. Nonetheless, can one dare conclude that it is impossible that electrical phenomena would be brought about by fluids, because one has not yet discovered the way in which these phenomena are produced? Doubtless no; we should not be discouraged because we have not been able to divine all the secrets of nature up to the present. The first sources may elude our researches forever, but in trying to divine them, we will not fail to make discoveries as they fall along the path.

§182 In this way, however difficult it may be to apply mechanical principles to physical effects, one must never abandon this manner of philosophizing, which is the only good one, because it is the only one with which one can make sense of the phenomena in an intelligible fashion. Doubtless one must not abuse it, nor, in order to explain natural effects mechanically, invent motion and matter as one pleases (which ordinarily, even in explanation, does not produce the desired effect), nor certainly without taking pains to demonstrate the existence of these matters and these motions. But neither must one limit nature to the number of fluids that we believe are needed for the explication of the phenomena, as several philosophers have done, and in particular Hartsoëker, who has chosen two kinds of elements, the first perfectly fluid and the second perfectly solid, to make sense of the phenomena, and who believes the world to be composed of these two types of matter, which he supposes to be inalterable. But Leibniz made him see that these two matters or elements are nothing but a fiction, contrary to the principle of sufficient reason, for the principle is the touchstone which distinguishes truth from falsity. Those who know the diversity that reigns in nature, and the admirable mechanism employed there, will not fix by a rash hypothesis the number and quality of the sources that she employs, rather, they admit only those the existence of which is demonstrated by experience or by solid reasoning.

§183 The littleness of the individual parts of matter surpasses so strongly anything that our senses could discover, that there is no hope that we could ever know their qualities, motion, and shape, which makes us see how far we are from the simple beings from which solid parts are formed.

§184 In this way, one is in error if one believes oneself to be able to make sense of the phenomena that fall under our senses by simple shape and the size of their sensible parts, because we do not know how many mixtures of primitive and irresolvable parts of matter were necessary, before the parts that fall under our senses had resulted. For as much as the matter of a body is composed of other matters mixed together, one must determine the difference between the parts of this body by the matters that compose it, and by the proportion in which they are mixed. In this way, if someone wanted to explain the effects of cannon powder, for instance, they would have to begin by determining of how many types of matter it is composed, and the proportion of their mixture, before passing to the shape of its parts. For the mixed matters, and their proportion, must precede mechanical causes (that is, the determination of the shape and the size of the parts), of which it is not permitted to speak until one has arrived at the primitive matters: these physical qualities, which make up the effect of mechanical causes, must necessarily precede them in the explanation of the phenomena.

§185 But as we are left with little hope of discovering the simpler matters, the mixture of which results in sensible bodies, a physicist who does not wish to waste his time must content himself with discovering the closest reasons that human industry can perceive, and will admit only matters and motions the existence of which can be demonstrated.

Translator's Notes

1. These notes were added by the editor for clarity and references to outside sources, and are not features of the original text.
2. "Esprit", in the original.
3. See Chapter 7 (translated in du Châtelet 2009/1750) for her discussion of matter more generally.
4. Newton called the magnitudes evaluated by his calculus, that is, magnitudes that increase or decrease continuously, "fluxion" magnitudes. The Newtonian calculus of fluxions is a calculus of quantities that flow in this sense (see Newton 1974 /1684–1691).
5. "Philosophes," in the original.
6. Du Châtelet is referring here to Leibniz's principle of sufficient reason. See du Châtelet 2009 / 1750, §8ff.; see also Iltis 1977.
7. Du Châtelet uses the term "sophisme." I have translated this "paradox," to follow the usual terminology, but also to avoid the pejorative meaning of "sophism."
8. See Salmon 2001, Black 1950, and Simplicius for discussion of this paradox.
9. See du Châtelet 2009/1750, §8ff. for this attempted proof.
10. "Quantité," in the original.
11. Du Châtelet probably is referring here to John Keill (1671–1721), a mathematician and Newtonian. The discussion of the properties of grains of sand is in Keill (1745), Lecture 5.
12. See Chapter 7, "The Elements of Matter," translated in du Châtelet 2009/1750.
13. It appears that du Châtelet means "actually resolvable" in this list, not "resolvable in principle." See §173 and §174 below.
14. Du Châtelet does not appear to mean that if a nourishing substance were divisible to infinity, then organisms that consume that substance would grow into larger or smaller members of their species, depending on how robust the nourishment was. Instead, she

appears to argue that organisms of the same species would develop differently and would become different species.
15. Converts it to glass.
16. Nicolaas Hartsoëker (1656–1725), the Dutch mathematician, inventor, and scientist.
17. "M. Volf" in the original. This is very likely a reference to Caspar Friedrich Wolff (1735–1794), a German physiologist.
18. Robert Boyle (1627–1691). A "graine" or grain is a pre-Revolutionary French unit of measurement equal to about 53 contemporary milligrams.
19. A "ligne" or line is a pre-Revolutionary French unit of measurement. A line is approximately 2.26 contemporary millimeters; there are 12 lines in a pre-Revolutionary, but not a contemporary, inch.
20. Réné Réaumur (1683–1756.)
21. §149, translated in du Châtelet 2009/1750.
22. Du Châtelet appears to refer here to hypotheses about the ether, which sometimes was supposed to be a fluid, and was employed in reasoning about electricity and magnetism.
23. Subtle matter is a term employed by Descartes, among others.

CHAPTER 22

Of the Idea of Necessary Connexion

DAVID HUME

Pages 155–172 of *A Treatise of Human Nature.* Book I, Section XIV. Edition reprinted from the original ed. in three volumes and ed., with an analytical index, by A. Selby-Bigge. Oxford: Clarendon Press, 1888/1739.

Having thus explain'd the manner, *in which we reason beyond our immediate impressions, and conclude that such particular causes must have such particular effects;* we must now return upon our footsteps to examine that question, which[1] first occur'd to us, and which we dropt in our way, *viz. What is our idea of necessity, when we say that two objects are necessarily connected together.* Upon this head I repeat what I have often had occasion to observe, that as we have no idea, that is not deriv'd from an impression, we must find some impression, that gives rise to this idea of necessity, if we assert we have really such an idea. In order to this I consider, in what objects necessity is commonly suppos'd to lie; and finding that it is always ascrib'd to causes and effects, I turn my eye to two objects suppos'd to be plac'd in that relation; and examine them in all the situations, of which they are susceptible. I immediately perceive, that they are *contiguous* in time and place, and that the object we call cause *precedes* the other we call effect. In no one instance can I go any farther, nor is it possible for me to discover any third relation betwixt these objects. I therefore enlarge my view to comprehend several instances; where I find like objects always existing in like relations of contiguity and succession. At first sight this seems to serve but little to my