Newton's *Principia* and philosophical mechanics

Katherine Brading*

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Abstract

Newton's *Principia* re-conceptualizes rational mechanics and physics, and offers a novel unification of these heretofore distinct disciplines. I argue for a reading of the *Principia* that insists on a strict distinction between the rational mechanics (in Books 1 and 2) and the physics (in Book 3), in which the Definitions and the Axioms/Laws play a surprising dual role that both distinguishes the rational mechanics from the physics and unifies them into a single project: a philosophical mechanics.

This offers a new angle on existing questions in the secondary literature, including the sense in which Books 1 and 2 are to be understood as "mathematical"; whether or not the *Principia* is a text in mechanics; why Newton came to adopt the dual label "Axioms, or laws of motion"; the epistemic status of the axioms; the relationship between the axioms and the Definitions; in what sense Book 3 is incomplete as a physics; and the problem of applicability.

1. Introduction

Newton's *Principia* re-conceptualizes rational mechanics and physics, and offers a novel unification of these heretofore distinct disciplines. In this paper, I argue for a reading of the *Principia* that insists on a strict distinction between the rational mechanics (in Books 1 and 2) and the physics (in Book 3), in which the Definitions and the Axioms/Laws play a surprising dual role that both distinguishes the rational mechanics from the physics and unifies them into a single project: a philosophical mechanics.

"Philosophical mechanics", as Marius Stan and I use that term, applies to projects that seek to combine rational mechanics with physics, as those enterprises were then understood. Looking back from our present-day vantage point, it can be difficult to appreciate the gulf that existed between these two, and the difficulties that were involved in bringing them together into a single project. My goal in this paper is to make vivid just how radical a re-conceptualization of mechanics and physics – and of the relationship between them – Newton's *Principia* achieves.

The value of this reading depends upon its utility in resolving interpretational puzzles and in bringing to light features of the *Principia* that otherwise lie hidden. These include the dual status of the Definitions and Axioms/Laws, the relationship of the mathematical to the physical, and the innovative reconceptualizations of mechanics and physics that make the *Principia* possible.

More generally, the value lies in the framework offered for re-thinking physics, mechanics, and natural philosophy in the 18th century. How we read Newton's *Principia* affects how we

* katherine.brading@duke.edu

understand its place in the wider philosophical landscape of the 18th century. At the beginning of the century, physics was a dependent subfield of philosophy, practiced by philosophers, whereas theoretical or rational mechanics fell under the authority of mathematicians (see section 2). By the end of the century, these disciplinary boundaries and domains of authority had been redrawn.[1] Stan and I argue that these transformations reflect an extended effort throughout the 18th century to provide a satisfactory philosophical mechanics of the material world, the failures and successes of which embody the central metaphysical, epistemological, and methodological lessons of natural philosophy in the age of reason.[2] By showing that Newton's *Principia* is powerfully understood within the framework of philosophical mechanics, this paper offers a first step towards re-evaluating physics and mechanics in the 18th century.

The main body of this paper is taken up in explicating and arguing for my interpretation of Newton's *Principia* as a text in philosophical mechanics. In section 2, I set out my view. I describe the rational mechanics (2.1), the physics (2.2), and then the philosophical mechanics (2.3) of the *Principia*. In section 3, I remove oversimplifications, discuss further considerations arising from the secondary literature, and address some puzzles present in the existing literature that my view solves.

2. Newton's *Principia*: a project in philosophical mechanics

My view is this. Newton's *Principia* combines rational mechanics (Books 1 and 2) with physics (Book 3). The *Definitions*, and the *Axioms, or Laws of Motion*, come prior to Books 1-3, and therefore pertain to both the rational mechanics and the physics. The result is a novel integration of rational mechanics and physics into a single project, a project in philosophical mechanics.

It can be hard to see why there should be anything of interest here, especially for any present-day reader who, turning to the *Principia*, sees simply a book in physics; even more so, for anyone who uses the terms "classical mechanics" and "classical physics" interchangeably. But this reaction rests on a conceptualization of the relationship between mechanics and physics that happened after Newton. So we first need to familiarize ourselves with the terms as Newton understood them, and as they were understood more widely at the time.

2.1 Rational mechanics

What is rational mechanics, according to Newton? In the Preface to the first edition of his *Principia*, published in 1687, Newton offered a taxonomy of mechanics in which he divided "universal mechanics" into three: practical mechanics, rational mechanics, and geometry. In so

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[1] Why, and how, did all this happen? And with what philosophical significance? One thing we know for sure is it is not, as older narratives suggest, that the scientific revolution happened in the 17th century, culminating in Newton's *Principia*, after which physics was independent of philosophy so that the 18th century was a period of working out the details and solving problems within "Newtonian mechanics" or "Newtonian physics". See also Brading and Stan, forthcoming.

[2] We make our case in Brading and Stan, *Philosophical Mechanics in the Age of Reason*, ms.

doing, he changed the domain of geometry and set out a bold new agenda for rational mechanics: the *exact* treatment of *any motions* under *any forces* whatsoever. To understand the epistemological status of rational mechanics, and its relationship to physics, we first need clarity about Newton's taxonomy of mechanics, which is as follows.[3]

Newton begins by noting that practical mechanics lacks the exactness pertaining to geometry. But this lack of exactness, he states, arises from imperfections in the mechanic rather than in the subject-matter of mechanics. A perfect mechanic, able to "work with the greatest exactness"[4], would be able to produce perfect circles and straight lines.

Moreover, the successful completion of this task is presupposed in geometry:[5]

> To describe straight lines and to describe circles are problems, but not problems in geometry. Geometry postulates the solution of these problems from mechanics, and teaches the use of the problems thus solved.

In other words, we presume the exact production[6] of the geometrical figures that we then use as the basis for developing the problems of geometry.[7] However, for Newton the means of production are not part of geometry; rather, the exact production of geometrical figures is the domain of what Newton calls "rational mechanics".

Rational mechanics is *exact* in the same sense that geometry is exact, but whereas geometry includes only *magnitude*, rational mechanics concerns also *motion*.[8] This is because rational mechanics concerns the production or generation through motion of the figures (curves, shapes, solids) whose properties are the subject-matter of geometry. Crucially, these motions arise from the application of forces, and there is no restriction to curves that are constructible by traditional methods. Rational mechanics, Newton says, is:[9]

> the science, expressed in exact propositions and demonstrations, of the motions that result from any forces whatever and of the forces that are required for any motions whatever.

We can sum up this conception as follows. On the one hand, like practical mechanics, but unlike geometry, rational mechanics considers motions and forces, and it considers any motions and

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[3] See also Garrison 1987; Domski 2003; Guicciardini 2009; Smeenk 2016; and references therein.
[4] Newton 1999, 381.
[5] Newton 1999, 382.
[6] I use the terms "production" and "generation" here, rather than "construction", because there is no requirement that the curves by constructible by traditional means. See Domski 2003 and 3.1.
[7] See the similar discussion Newton offers in his treatise on geometry, as discussed and quoted in Guicciardini 2009, 300. Mechanics is the means by which the objects of geometry are produced, and geometry presupposes that these objects have been so exactly.
[8] Guicciardini 2009, 298.
[9] Newton 1999, 382.

forces whatsoever. On the other hand, like geometry and unlike practical mechanics, rational mechanics is exact.[10]

This taxonomy of universal mechanics differs from that of Newton's contemporaries in the relationships it describes between geometry and mechanics.[11] Moreover, this conception of universal mechanics alters the domains of both geometry and rational mechanics. Newton *extended* the subject-matter of geometry beyond that admitted by Descartes and others, by including curves produced by any motions resulting from any forces.[12] He *restricted* the domain of geometry by placing the problems of generation in the domain of mechanics. And he *extended* the domain of mechanics beyond the mechanical powers associated with the five machines (the lever, pulley, winch, wedge and screw of Archimedes) to incorporate *natural* forces:[13]

> But since we are concerned with natural philosophy rather than the manual arts, and we are writing about natural rather than manual powers, we concentrate on aspects of gravity, levity, elastic forces, resistance of fluids, and forces of this sort...

This incorporation of natural forces and motions within the domain of rational mechanics is, for our purposes, the most important philosophical move that Newton makes in setting out his new taxonomy. For it is this that connects rational mechanics to physics, in a specific way that we discuss below.

Books 1 and 2 of the *Principia* are books in rational mechanics in the above sense: they provide an exact treatment of the generation of curves by means of forces. In them, Newton is attempting unprecedented genenality: no matter what motions we consider when we turn our attention to the physical world, the mathematics to treat those motions in terms of forces is to be found in rational mechanics.

However, as books in rational mechanics they are incomplete: they lack any definitions and axioms from which the demonstrations in Books 1 and 2 are to proceed. For these, we turn outside Books 1 and 2, for it is prior to these books that Newton gives us his *Definitions* and *Axioms, or Laws of Motion*. In my view, with respect to Books 1 and 2, these definitions and axioms/laws are to be read as principles of rational mechanics. The reason that they are placed outside Books 1 and 2 is that they pertain to all three books of the *Principia*, as we will see in more detail below.

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[10] In Cohen and Whitman's translation of the *Principia*, Cohen writes in a footnote (Newton 1999, 381) that "Newton's comparison and contrast between the subject of theoretical or rational mechanics and practical mechanics was a common one at the time of the *Principia*", citing a later reference (from 1704) that in turn appealed to the authority of John Wallis. However, there is a great deal more going on in this Preface than that distinction. For example, Guicciardini (2009, 294) *contrasts* Newton's view of the relationship between geometry and mechanics with that of Wallis, writing that Wallis defined mechanics as an *application* of geometry to the science of motion. As we have seen, this was not Newton's conception.

[11] See Domski 2003 and Guicciardini 2009.

[12] Guicciardini (2009) emphasizes that Newton's philosophy of geometry extends geometry beyond Descartes's account. Crucially, Descartes excluded "mechanical curves from the realm of the exactitude and certainty of geometry" (*ibid*, 299). For Newton, what matters for the purposes of geometry is that the resulting objects are exact, not the means by which they are constructed (*ibid*, 301).

[13] Newton, 1999, p. 382.

By opening his work with a set of definitions followed by a set of axioms, Newton structured his rational mechanics analogously to standard presentations of Euclidean geometry from the period. In so doing, he indicated the epistemic status to be accorded to the definitions and to the axioms, the standards that demonstrations in rational mechanics are expected to meet, and the appropriate criteria of justification for the claims made by rational mechanics. I will return to the relationship between the definitions and the axioms, and to their epistemic status, below. For now, I wish to highlight a different issue: the *character* of the mathematical challenge faced by Newton, to which his rational mechanics is a solution. De Gandt (1995) puts the issue beautifully in his discussion of Wren, Hooke, Halley, Newton and the search for a proof that the celestial motions are derivable from an inverse square force law. He writes:[14]

> Wren required truly deductive demonstrations, and what Hooke provided did not convince him. But what would a demonstration in these matters be like? What examples could be used? … In geometry, the criteria of proof were well established by virtue of a culture nourished by the books of the ancients and cultivated in discussions, courses, challenge contests, and discoveries. But what would constitute proof when it was a question of forces and motions? What indubitable principles could be adopted as a foundation? What mathematical tools should be used?

Geometry, with its long history of development, had achieved a stability of method: a wide range of problems were solved and soluble by means of the same techniques, meeting the same standards of demonstration.[15] As yet, there was no science that connected forces and motions such that a variety of different problems were soluble under the umbrella of a single set of principles, methods, and standards.[16] In manuscripts from the decades preceding the *Principia*, we see Newton wrestling with this challenge as he sought to solve problems of increasing generality,[17] and in the rational mechanics of the *Principia* he offered his developed response.

We now have on the table the main elements of Newton's conception of rational mechanics. Thus far, I have used the terms "mechanics" and "rational mechanics" as if they are unproblematic terms. However, Newton's *Principia* played a crucial role in the reconceptualization of mechanics that began prior to the *Principia* and achieved an explicitly articulated form in Lagrange's

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[14] De Gandt 1995, 6.

[15] For example, Newton goes to great lengths to use (as far as possible) long-standing geometrical methods for his demonstrations in the *Principia*, notwithstanding being at the forefront of developments in mathematics. See also Landry "Mathematics: method without metaphysics" ms., who argues that, since the time of Plato, it is precision of definitions and stability of method that underwrites the practice of mathematics.

[16] As a simple illustration of this point, note that geometry considers magnitudes, but not the *directed* magnitudes required by problems involving forces and motions. So new rules of how to combine (i.e. to add, subtract, and so forth) directed magnitudes had to be introduced and justified.

[17] This account of Newton's work prior to the *Principia*, in which we see him striving to solve problems of increasing generality and through this process developing the conceptual innovations and resources that we find in the *Principia*, is due to Solomon, 2017.

Mecanique Analitique at the end of the 18th century. In addressing whether or not Newton's *Principia* is a treatise in mechanics, Gabbey (1992) emphasizes that the term was used in different ways at the time Newton was writing, and that we cannot assume that our contemporary usage reflects a category applicable to Newton's treatise.[18] He says:[19]

> Mechanics as a discipline underwent radical changes in nature during the period covered by the prehistory, publication and early reception of the *Principia*. Correspondingly, the term "mechanics" as used by writers of the time often carried equivocal senses.

In light of these complexities, it is important to be careful how we handle the term "mechanics". Moreover, we also have the 17th century rise to prominence of the "mechanical philosophy": an approach to natural philosophy that was largely qualitative, at least in Descartes's hands, and distinct from the mathematical discipline of mechanics (more on this below, when we turn our attention to physics). So there is ample opportunity for confusion and misunderstanding surrounding the term "mechanics". Nevertheless, I think that Newton is clear in his conception of rational mechanics as he presents it in the *Principia*, and that Books 1 and 2 realize this conception. It is a discipline of mathematics, and it concerns the exact treatment of any motions under any forces whatsoever.

Rational mechanics provides us with one strand of Newton's project in philosophical mechanics. The second comes in Book 3, which is a book in physics. Once we have both strands in place, we will be in a position to see what is at stake and why all of this matters.

2.2 Physics

When the *Principia* was published, Newton faced accusations that he had produced a book in mechanics, but not in physics.[20] As noted above, this is puzzling to any reader today who, turning to the *Principia*, sees a book in physics, and even more so to one who uses the terms "classical mechanics" and "classical physics" interchangeably. But that very reconceptualization of mechanics in relation to physics happened after Newton. It was not in place at the time Newton wrote his *Principia*, nor for some time afterwards.

We can begin with Rohault's *Traité de physique*, published in 1671, as an exemplar of how the term "physics" was understood at the time. Rohault opens with a chapter entitled "The Meaning of the Word Physics, and the Manner of treating such a Subject", in which he says that "we here use it [the word physics] to signify Knowledge of natural Things, that is, that Knowledge which

[18] For more on Gabbey's discussion of the *Principia* as a treatise in mechanics see 3.3.
[19] Gabbey 1992, 308.
[20] See the review published anonymously in the *Journal des Sçavans* (2 August 1688) 153ff, and the English translation in Koyré 1965, 115.

leads us to the Reasons and Causes of every Effect which Nature produces".[21] Physics, thus understood, encompasses all "natural Things", including all of their causes and effects.

This scope for physics persists into the 18th century. Musschenbroek's early 18th century characterization of physics is, I think, particularly helpful. Physics, he says, is that part of philosophy which "considers the space of the whole universe, and all bodies contained in it; enquires into their nature, attributes, properties, actions, passions, situation, order, powers, causes, effects, modes, magnitudes, origins"[22]. Excluded from physics are the study of spiritual beings, teleological investigations of things, metaphysics (which studies "such general things as are in common to all created beings", both physical and spiritual), moral philosophy and logic. For our purposes, the most important things to note are the inclusion of physics within philosophy, and the characterization of the domain of physics. According to Musschenbroek, the primary subject-matter of physics is bodies, and many aspects of bodies that we might think of today as being the subject-matter of metaphysics – such as the nature, powers, causes, effects and origins of bodies – fall within physics. This conception of physics was widely held at the time, both before and after the publication of Newton's *Principia*.[23]

The term "physics" was often used interchangeably with "natural philosophy", and the pursuit of physics fell within the remit of philosophers. Mechanics, on the other hand, was a distinct discipline, practiced by mathematicians. Gabbey (1992, 310) emphasizes that during the Renaissance and early seventeenth century, works in mechanics and in physics were written by distinct groups of authors between which there was little overlap. Even where there is overlap, the two subjects were treated largely separately.[24]

Rohault is an example of this: he presented his mechanics in a separate treatise from his physics.[25] The subject-matter of Rohault's mechanics is machines (artefactual devices), whereas that of his physics is the natural world. The two treatises proceed from different principles, by means of different methods, and with different goals. Moreover, in the Preface to his English translation of Rohault's mechanics, Watts describes mechanics as an application of geometry

[21] My quotations are from the English translation: Rohault 1723, 1.
[22] Musschenbroek 1744, 1-2.
[23] See Brading and Stan, forthcoming, and *Philosophical Mechanics in the Age of Reason*, ms.
[24] Though mechanics and physics were largely separate there were active attempts to bring them together, and there is a long history of "mixed mathematics". Famously, Galileo expanded the range of problems treated in mechanics and sought a mathematical theory of natural phenomena. In his case, he eschewed talk of causes, and therefore of physics as it was then understood. Kepler, on the other hand, transformed positional astronomy into physical astronomy via his commitment to a causal explanation of the motions of the planets and his demand that the mathematical theory and the causal explanation align exactly. And Huygens, with greater success than anyone else, took Descartes's qualitative "unification" of matter theory and mechanics seriously and used it to attempt a philosophical mechanics. There is more to be said about each of these cases, but the simplified picture is as follows. In mechanics, physical bodies were taken as given, abstracting only those properties (such as size, shape, and weight) relevant to the demonstrations. Mechanics could assume the existence of such bodies because another discipline – physics – was tasked with the general theory of bodies (of their nature, properties, and so forth). See also 2.3.
[25] Rohault's *Mechanics* (*Les mécaniques*) was published posthumously in 1682. An English translation by Thomas Watts was published, the second edition of which (Rohault, 1717) is my source. The full title reads: *A Treatise of Mechanics: Or, The Science of the Effects of Powers, or Moving Forces, as apply'd to Machines, demonstrated from its first Principles*.

remarking that these applications are *unaffected by the Cartesian philosophy* that seems at times to be presupposed. He writes:

> If our Author in his Definition of Gravity, or in an Expression or two besides, seems to refer to the *Cartesian* Philosophy, now deservedly exploded, yet it is done in such a Manner, as does not in the least affect the Demonstrations, which will be equally true, whatsoever Hypothesis we follow in those Points.

In other words, Rohault's mechanics is independent of his physics.

In short, physics, or natural philosophy, studied physical bodies including their natures, properties, causes and so forth. This was the work of philosophers. Mechanics, on the other hand, studied the behavior of machines, and belonged to the domain of the mathematicians. In the mid to late 17th century, notwithstanding Descartes's "mechanical philosophy", the integration of mechanics and natural philosophy into a single discipline did not yet exist.

Books 1 and 2 of Newton's *Principia* explicitly eschew consideration of the physical causes and effects of the motions of bodies, considering these issues only "mathematically". Newton makes this clear multiple times (for example, in the opening paragraph of Book 1, Section 11) and re-emphasizes the point at the beginning of Book 3 when he writes:[26]

> In the preceding books I have presented principles of philosophy that are not, however, philosophical but strictly mathematical.

While Books 1 and 2 offer a radical expansion of the domain of mechanics to include natural forces and motions, they are mathematical, and they do not consider causes. As I argued above, they are rightly considered books in mechanics. My point here is that they are not books in physics, as physics was then conceived.[27]

Nevertheless, Newton did intend the *Principia* to include physics. In the scholium to Proposition 69 of Book 1 of the *Principia*, Newton explains how he understands the relationship between rational mechanics and physics. Whereas rational mechanics is the exact mathematical treatment of any forces and motions whatever, physics considers the actual motions of bodies and the actual forces responsible for those motions. Moreover, physics is also concerned with the causes of those motions. In proceeding from rational mechanics to physics, we first determine which forces are actual (i.e. which of the force laws explored by rational mechanics pertains in the behaviors of actual bodies), and then we seek the causes of these forces.[28] So the method has three

[26] Newton 1999, 793.
[27] I simplify in presenting my thesis, and in two important respects. First, Newton is unfolding his rational mechanics with an eye to the physics he hopes to treat. This is no accident, of course, given the genesis of the *Principia*. Second, in various scholia in Books 1 and 2 he illustrates the applicability of his mechanics with physical examples. Neither of these caveats undermines the central thesis that Books 1 and 2 are books in rational mechanics. See 3.5 for detailed discussion of this point.
[28] See 3.4 for Newton on causes and the relationship between forces and causes.

steps: develop the general theory of forces and motions (rational mechanics); determine which forces are actual (physics); identify the causes (physics). This is crucial for understanding the successes and limitations of the *Principia*, as we shall see.

If we look at the scholium in more detail, we see that Newton begins by re-emphasizing the point that (at least in Book 1 of the *Principia*) he is giving a "mathematical" treatment of "force", "attraction" and "impulse"; that is to say, he is treating "not the species of force and their physical qualities", but the "quantities" of force and their "mathematical proportions". He then writes:[29]

> Mathematics requires an investigation of those quantities of force and their proportions that follow from any conditions that may be supposed. Then, coming down to physics, these proportions must be compared with the phenomena, so that it may be found out which conditions of forces apply to each kind of attracting bodies. And then, finally, it will be possible to argue more securely concerning the physical species, physical causes, and physical proportions of these forces. …

Cushing says of this passage:[30]

> That is, there are three different levels at which we must work: the mathematical (or deductive), where we analyze the implications of certain assumptions or axioms; the physical, where we use comparison with data to decide which of the many possible axioms or laws actually do correspond to nature; and, finally, the philosophical, where we seek the causes of these laws. In the *Principia*, he attempted to do the first two as a preparation for the third that he also felt to be important. … In his life Newton never did succeed in constructing an explanation of the causes behind his laws of mechanics and of gravitation.

I agree with the overall message here about the three steps. The first lies in rational mechanics.[31] The third concerns causes, and – as we have seen – this places it within the domain of physics, as physics was conceived at the time. Cushing's label of "philosophical" for the third step is appropriate in the sense that physics, as it was then understood, was that part of philosophy that included the search for the causes and effects of the properties and behaviors of bodies. It is this third step in Newton's three-step methodology that his contemporaries would have recognized as physics.

For Newton, however, physics involves a prior step. His second step, in which we treat physical phenomena using the results developed in our rational mechanics, is also explicitly located within physics. This is what Newton does in Book 3. Specifically, he treats the force of gravity as a physical force. He uses the mathematics of Books 1 and 2 to theorize gravitational phenomena,

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[29] Newton 1999, 588-9.

[30] Cushing,1998, 95.

[31] I disagree with Cushing that Newton's axioms can be understood as assumptions, freely chosen. I discuss their epistemic status in 2.3.

both terrestrial and celestial, with spectacular success.[32] While Books 1 and 2 of the *Principia* are books in rational mechanics, Book 3 is – for Newton – a book in physics.

Famously, however, Newton does not uncover the causes of the force of gravity. As he himself states in perhaps the most notorious passage of the *Principia* (added in the second edition of 1713):[33]

> Thus far I have explained the phenomena of the heavens and of our sea by the force of gravity, but I have not yet assigned a cause to gravity. … I have not as yet been able to deduce from phenomena the reason for these properties of gravity, and I do not feign hypotheses.

This claim, and others like it, were part of a widespread dispute at the time over whether in the *Principia* Newton had indeed provided a physics. The difficulties were all but inevitable because of the prevailing conception of physics: it was that part of philosophy charged with providing *causal* knowledge of the natural world, and in particular of the behaviors of bodies. By Newton's own admission, the *Principia* begins but does not complete this task, and this left evaluation of what the *Principia* achieves unclear to many of his contemporaries. As the first reviewer of the *Principia* complained (see above), Newton seemed to him to have provided a perfect mechanics, but to have failed to provide a physics, for he had failed to provide a complete account of the *causes* of the gravitational behavior of bodies.

However, read in the context of the scholium to Proposition 69 of Book 1 (quoted above), Newton's meaning is clear. The achievements of Book 3 lie in step 2: he had completed step 2 but not step 3 of his three-step methodology. This is an achievement in physics: Newton had determined that there *is* a gravitational force acting among bodies, and what conditions that force satisfies. Step 2 contributes to our causal account of the world, and properly belongs to physics.[34]

Nevertheless, it is easy to see why Newton's three-step methodology was a source of great confusion among his contemporaries and 18th century critics.[35] Step 3, had Newton completed it, would have been recognizable to his contemporaries as physics. Placing Step 2 within physics was novel. Newton offered it as a bridge linking rational mechanics to traditional physics, and then made this bridge a part of physics: physics, in Newton's reconceptualization, begins not with the qualities of bodies, but with the mathematical treatment of the motions of physical bodies in terms of forces. With or without Step 3, this is a radical transformation of the goals and methods of physics.

From our vantage point, we tend to use the terms "classical physics" and "classical mechanics" interchangeably, and to think of mechanics as a part of physics. I have emphasized that this is not

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[32] My account of Newton's three-step methodology differs from that offered by Cohen in his "Newtonian style". See 3.4 and 3.5.

[33] See Newton 1999, 843.

[34] For Newton on forces and causes and the complexities in the relationship between them, see 3.4.

[35] On the incompleteness of Newton's causal account of gravitation see Janiak 2008, ch. 3, especially p. 64, and Biener 2018, 3, and Janiak's discussion therein of Leibniz's and Clarke's attempts to grapple with the issue.

how things stood during Newton's lifetime. Instead, it is appropriate to view the *Principia* as offering a conjunction of two books in mechanics with one book in physics, where that book in physics is, by the author's own criteria, incomplete. In so doing, we are able to see the highly revisionary conceptions of each that Newton offered. Moreover, it is not just that Newton transformed mechanics and physics individually, he also transformed the relationship between them. All three books of the *Principia* are gathered together into a single text, and the significance of this is our next concern.

2.3 Philosophical mechanics

We have seen that, as of the late 17th and early 18th centuries, rational mechanics belonged to mathematics whereas physics was a subfield of philosophy. In our book, Stan and I argue that, from the late 17th through the 18th century, the guiding research program for much of natural philosophy was what we call "philosophical mechanics".[36] Projects in philosophical mechanics seek to integrate rational mechanics and physics into a unified treatment of bodies and their motions. In what follows, I argue that Newton's *Principia* is fruitfully understood as offering a philosophical mechanics.

Newton uses the term "natural philosophy" for his project, as the full title of the *Principia* makes clear and as he states in the Preface. However, he is also explicit that he intends this to include both rational mechanics and physics. Since the terms "physics" and "natural philosophy" were often used interchangeably at the time, the term "natural philosophy" risks masking the very distinction I have highlighted between rational mechanics and physics.[37] This is one reason why Stan and I chose to adopt the term "philosophical mechanics". It is a term of art, not found in the literature until the late 18th century, and even then with a meaning not quite as we use it.[38] We introduce it for the conceptual work that it enables us to do as we work through developments in 17th and 18th century natural philosophy, physics, and mechanics.

We have seen that Books 1 and 2 of Newton's *Principia* are books in rational mechanics, whereas Book 3 is a book in physics. In addition to Books 1-3, the *Principia* contains a Preface followed by a set of *Definitions* and then three *Axioms, or Laws of Motion*. These precede Books 1-3. These are the main elements that Newton assembles into a single text under the title

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[36] *Philosophical Mechanics in the Age of Reason*, ms.

[37] From Newton's correspondence with Halley, it is clear that his choice of title deliberately reflects the expansion in scope from Books 1 and 2 (developed in the "De Motu" manuscripts), of most interest to mathematicians, to include Book 3, of most interest to natural philosophers. See also Newton's introduction to Book 3 of the *Principia*, where he describes the first two books as not "philosophical" but instead "strictly mathematical", noting also that in these two books he illustrates the applicability of his mathematical results to actual bodies and motions in several scholia. For Book 3, it "remains for us to exhibit the system of the world from these same principles". Here, we think that the framework of philosophical mechanics is helpful in clarifying Newton's intended distinction between the "mathematical" work of Books 1 and 2 (rational mechanics) and the "philosophical" work of Book 3 (physics, as that term was understood at the time).

[38] While the term was first used (to our knowledge) in a French text on mechanics by Prony (1800), Stan and I adopt it for our own purposes. See Brading and Stan, *Philosophical Mechanics in the Age of Reason*, ms., for further discussion.

Mathematical Principles of Natural Philosophy. The structure of how they are put together is significant. First, it indicates that Newton conceived of his *Principia* as a unified project. Second, it shows that the Preface, the *Definitions*, and the *Axioms or Laws of Motion*, pertain to *all three books*: they are the hinge that joins Books 1 and 2 to Book 3, creating a unified whole.

Conceiving of the project in this way provides insight into the status of the *Definitions* and of the *Axioms, or Laws of Motion*. Newton's placement of these elements prior to and outside the three books of the *Principia* is surely deliberate.[39] In the context of rational mechanics, the *Axioms or Laws of Motion* have the status of axioms, whereas in the context of physics, they have the status of laws of motion.[40] This gives them a highly interesting character. On the one hand, in their different roles they differ in their justification. On the other, as a single set of principles they unify rational mechanics and physics in a very special way. I discuss both these aspects below.

First, however, notice how surprising this unification is. Descartes, in his physics, offered us his laws of nature. Nowhere in this physics does he offer us a mechanics, or provide axioms of mechanics.[41] Rohault, as we noted above, wrote two separate treatises, one on physics and one on mechanics. Both texts contain axioms, one set for physics and another for mechanics, but the axioms are utterly different. Rohault's axioms of physics begin:[42]

> The first is, that Nothing, or that which has no Existence, has no Properties…
>
> Secondly, It is impossible that something should be made of absolute Nothing; or that mere Nothing can become any Thing…

Whereas his first axioms of mechanics are:[43]

> Ax. 1. In heavy Bodies, which are Regular and Homogeneous (that is, which have all their Parts equally heavy) and plac'd Horizontally, the Center of Magnitude is also the Center of Gravity
>
> Ax. 2. The different Gravities of Homogeneous Bodies are one to another in Proportion to their Bulks

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[39] Biener, 2018, opens by saying that the *Axioms, or Laws of Motion* are in Book 1, but I think this is not right. He then says that the placement of the Rules of Reasoning at the beginning of Book 3 mirrors this positioning, but he doesn't say very much about why we should think this, and I do not agree. In my opinion, the *Axioms, or Laws of Motion* are very deliberately placed prior to Books 1-3, and the *Rules of Reasoning* are similarly deliberately placed within Book 3. See section 3 for further discussion of the status of the *Axioms, or Laws of Motion*.

[40] Adopting such a clean terminological distinction is conceptually helpful but oversimplifies the situation. For a more nuanced treatment, see 3.2.

[41] This notwithstanding the "mechanical philosophy" offered in his *Principles*, see above. Descartes does not use his laws of nature to develop a mechanics in the sense intended here: his laws are not principles from which he mathematically demonstrates the behavior of machines.

[42] Rohault 1723, 18-19.

[43] Rohault 1717, 5.

Newton's use of a common set of axioms for both mechanics and physics is striking.

In the seventeenth century, works in mechanics and physics were most often written by different people, and the two subjects were treated largely separately, as Gabbey has discussed (see above). Nevertheless, as Gabbey goes on, discussion over the appropriate taxonomy for the materials treated under the umbrellas of mechanics and physics, and how these materials related to one another, was live and explicit. Newton was exposed to this not least through the lectures and writings of Isaac Barrow, John Wallis, and Robert Boyle, in which the mathematical principles of mechanics find a new place in the foundations of natural philosophy.[44] So Newton's unification does not come out of nowhere, and should be viewed in the context of this wider community project. It is the creativity and depth of Newton's contribution that stands apart.

Newton's use of the label "Axioms, or Laws of Motion", rather than simply "Axioms" or "Laws" is significant, for it provides information about the dual role that these principles are required to play. In labelling his principles "Axioms", Newton followed a standard practice in mechanics. Moreover, he structured his rational mechanics analogously to geometry, thereby providing information about the epistemic status of the *Axioms*. We will return to this important point below, but first a few words about *Laws*.

It is well-known that in Descartes's philosophy laws of nature play an important role, situated as they are between his metaphysics and his physics, providing a bridge between an immutable God and the changing world. However, while laws of nature are an important theme of 17th century natural philosophy, whether or not to include such laws, and if so their formulation, their appropriate placement within a philosophical system, and their metaphysical and epistemic status, remained open questions. For example, Descartes's laws of nature have no special place in Rohault's *Traité de physique*, and indeed only the first law receives this label in Rohault's exposition.[45] Nevertheless, it is clear from Newton's manuscripts that Newton's own laws of motion have their origins in Descartes's laws of nature, and it is also evident that Descartes's *Principles of Philosophy* was both inspiration and target for Newton's *Mathematical Principles of Natural Philosophy*. In retaining the terminology of laws, Newton is signalling two things. First, that his three principles pertain not only to mechanics, but also to physics. Second, that they play a role in his physics similar to that which Descartes's laws play in his: specifically, just as Descartes aspires to a deductive structure for his physics, at least in principle, so too Newton's laws lie at the basis of deductive arguments in his.

The dual role of Newton's *Axioms, or Laws of Motion* in his mechanics and physics affects their epistemic status and justification. As axioms, they have the status of mathematical hypotheses in the specific sense that our demonstrations assume them as given and proceed from there. That is, taking them as axioms allows us to proceed as if they are true.[46] The wider the class of problems

[44] Gabbey 1992, 311-14.

[45] Rohault does not present Descartes's three laws of nature as laws, but he does write (1723, p. 47) that "it is one of the Laws of Nature, *that all Things will continue in the State they once are* unless any external Cause interposes", which is the beginning of Descartes's first law of nature.

[46] See De Risi (2016) for discussion of different epistemological approaches to the axioms of geometry in the early modern period. Here, I gloss over those differences: the important points are that axioms are (i) taken as true and (ii)

falling under the scope of the axioms (that is, soluble via mathematical reasoning from the axioms), the greater our justification for adopting them as axioms for rational mechanics. In short, their justification lies in their *generality*.[47]

As laws of motion, on the other hand, they have the status of empirical claims: they are claims about the physical world that may or may not be true. The wider the range of phenomena to which they may be successfully applied, the greater our justification for taking them to be true. In short, their justification lies in their *universality*. Universality of applicability justifies their status as laws of motion. As laws of motion, they rest on an inductive basis: as Newton is at great pains to emphasize in his scholia to the *Axioms, or laws of motion*, the laws are empirically well-supported.[48] And so, we take them to be true "until yet other phenomena make such propositions either more exact or liable to exceptions".[49] The standards and criteria for successful application to the phenomena are set by the community of physicists, just as the standards and criteria for successful solutions of problems in rational mechanics are set by the community of mathematicians.

This dual justificatory status for a single set of principles makes clear that Newton's *Axioms, or Laws of Motion* unify rational mechanics and physics in a very special way. They are justified twice over and independently, first within rational mechanics and second within physics, and they thereby form part of the hinge that joins rational mechanics to physics. They are a conduit through which rational mechanics may speak about physics, and through which physics may pose problems for rational mechanics.[50]

The *Axioms, or Laws of Motion* do not, of course, stand alone. They are preceded by the *Definitions*. These too, fall outside Books 1-3, and pertain to them all. I have often puzzled over the relationship between the definitions and the laws of motion. There is so much overlap between Definition 3 and the first law of motion, for example, so why did Newton need both? And why is one a definition and the other an axiom or law of motion? I think we can solve this by viewing the *Definitions* and *Axioms, or Laws of Motion* through the lens of philosophical mechanics.

Consider first rational mechanics, understood as an exact science analogous to geometry. De Risi (2016, 602) notes that in the early modern period many, though not all, mathematicians

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required to meet appropriate conditions of justification. See Landry "Mathematics: method without metaphysics" ms. for elaboration of an "as if" interpretation of mathematical hypotheses. In early drafts Newton used the term "hypotheses" and then changed this for "laws". See 3.1 for more on the epistemic status of the axioms, and below for the relationship between the definitions and the axioms.

[47] Cohen (1980, p. 101) in his discussion of Newton's methodology, the "Newtonian style", highlights the importance for Newton, as a mathematician, of generalization.

[48] Though on this point see sections 3.2 and 3.6. Friedman (2001) takes this claim by Newton to be disingenuous and his constitutive interpretation of the laws offers an alternative account of their universality (see 3.6). For Descartes, the laws are similarly universal in their applicability, but their justification is *a priori*, depending upon prior accounts of the nature of matter and motion, and on the nature of God. For more on applicability, see 3.5.

[49] Newton 1999, 796. Newton allows that other regions of our universe may be subject to different laws. Nevertheless, Newton's method encourages us to take the laws as true throughout the universe until the phenomena show us that we must restrict their scope. See Smith (2014). See also Biener and Schliesser 2017, 317.

[50] This dual status ensures that the applicability of rational mechanics to problems in physics is not piecemeal, one problem, system, or scenario at a time; the shared axioms guarantee a (partially) shared modal structure for the problem space of rational mechanics and physics (see 3.5).

believed that the axioms of geometry were provable from the definitions, even when no such proof was explicitly given. It was the definitions that were considered the basic principles of mathematics. The postulates and axioms were statements, derivable from the definitions, to be used in the ensuing proofs. If we follow this approach then Newton's axioms should, in principle, be derivable from his definitions. However, even with this derivation in place, a problem remains: the definitions are neither self-evident nor do they rest on convincing empirical evidence. As such, they lack the epistemic status from which to derive a system of rational mechanics. The axioms, on the other hand, enable the solution of an unprecedentedly wide-ranging and general set of problems, as Book 1 demonstrates. This justifies their status as axioms, and confers justification on the definitions themselves insofar as they are necessary for the statement of the axioms. The status of the definitions as basic principles is legitimate insofar as the axioms are derivable from them. Thus, the axioms play an intermediary role between the definitions, whose status they justify, and the ensuing system of rational mechanics, whose mathematical derivation they facilitate.

The relationship between the *Definitions* and the *Axioms, or Laws of Motion* is rather different in the context of physics. To see this, we first need to consider how the *Definitions*, like the *Axioms, or Laws of Motion*, have a dual face. This is perhaps obvious, since the *Definitions* are clearly to be read in either a mathematical or a physical key, as appropriate. For example, the terms "body" and "force" can be taken to apply to physical bodies (e.g. "snow" in the explanation of Definition 1) and physical forces (e.g. "magnetic force" in the explanation of Definition 5), as well as to bodies and forces considered not physically but mathematically (e.g. in the discussion of Definition 8, where Newton is explicit about this). I understand every definition to have a dual reading, as either mathematical or physical, depending on whether the problem to be solved is a problem in rational mechanics (as in Books 1 and 2) or in physics (as in Book 3).

Particularly important, I think, is the dual face of the term "measure". Mathematically – in rational mechanics – when a quantity measures (an aspect of) something, it allows for its treatment as a magnitude: that is, for its treatment within the science of geometry. The role of the definitions is to set up those magnitudes that are then to be treated using the tools of geometry. (This point, that traditionally geometry was the science of magnitude, will be important in a different context later on, in section 3, when we return to the issue of the status of Newton's axioms, and need to recall that geometry was not, at the time, always assumed to have *space* as its subject-matter.) For example, in introducing "quantity of motion" as a measure of motion in Definition 2, Newton tells us how to treat motion as a magnitude. In so doing, he sets up the conditions for motion to be treated using the tools of geometry.[51]

In the context of physics, the role of the *Definitions* is to articulate the relationships among the relevant quantities and parameters to which we have empirical access through measurement. For example, Newton relates "quantity of matter" to two quantities whose empirical measurement is

[51] There is a sleight of hand here. Newton's geometry requires the treatment of (to use anachronistic terminology) vector quantities as well as scalar, and so new rules for the addition of such quantities must be introduced. The debate over the status of the parallelogram rule for the addition of forces is a prominent example of where this issue arises (see, for example, Miller 2017 and references therein).

already well-established (density and volume), and he relates "quantity of centripetal force" to motion and time. In so doing, he sets up the conditions for mass and centripetal force to be treated through quantitative empirical measurements.

In this way, the dual face of the term "measure" allows the *Definitions* to function successfully in both rational mechanics and empirical physics. Though a simple point, I think this is enormously important for understanding what Newton achieves in the *Principia*, and how he achieves it. In his lectures on Newton, George Smith has repeatedly emphasized the importance for Newton of quantities and their empirical measure, and Newton's remarkable ability to transform seemingly intractable puzzles about the natural world into problems that he could handle mathematically. Viewed through the lens of philosophical mechanics, we gain important insight into the philosophical moves that make this possible. Newton unifies rational mechanics with physics, not by collapsing them into a single discipline, but via a dual role for the *Definitions* and the *Axioms, or Laws of Motion*. Rational mechanics and physics each retains its own subject-matter, but through the *Definitions* and *Axioms, or Laws of Motion* the results of the former are applicable in the domain of the latter, and the problems of the latter fall within the domain of the former. It is in this sense that they are *mathematical* principles of *natural philosophy*.

For those who have emphasized Newton's claims that Book 1 is a book in mathematics, the problem of applicability has loomed large: what justifies the move from mathematics to physics? A variety of responses is available in the literature (see for example Cohen, 1980; Ducheyne, 2012, pp. 79-80; Smeenk, 2016). My own reaction was, for a very long time, enormous puzzlement over what the problem was supposed to be, since (to my mind) Book 1 was clearly a book of physics – recognizably so from any modern-day training in physics – in which physical subject-matter is discussed in the language of mathematics, with no special problem of applicability. But this is not right. Neither geometry nor rational mechanics have physical entities as their subject-matter. Rather, it is through the dual aspect of the principles – the *Definitions* and the *Axioms, or Laws of Motion* – that we are able to use the language of rational mechanics to talk about the subject-matter of physics. These principles sit outside Books 1, 2 and 3, serving both the rational mechanics and the physics; it is this dual aspect that provides the bridge from the rational mechanics to the physics, and that ultimately unifies the two.[52]

There is, however, an important contrast between the scope of Books 1 and 2 as compared to that of Book 3. Newton intended generality for his rational mechanics. He sought a rational mechanics with resources appropriate for *whatever* forces and motions are found in the world, whether in manmade machines or natural bodies. This scope for Newton's rational mechanics contrasts sharply with that for his physics. The focus of Book 3 of the *Principia* is one force: gravitation. As a result, what Newton provides is an attempted philosophical mechanics of gravitation.

To sum up. The framework of philosophical mechanics is helpful in clarifying Newton's intended distinction between the "mathematical" work of Books 1 and 2 and the "philosophical" work of Book 3, and the relationship between them. Books 1 and 2 are books in rational mechanics;

[52] For more on applicability, see 3.5.

Book 3 is a book in physics; and the three books are powerfully unified into a single system via the shared *Definitions* and *Axioms, or Laws of Motion*. This unification allows resources from Books 1 and 2 to be deployed in Book 3. More generally, the unification allows rational mechanics to be applicable within physics, and it allows physics to present rational mechanics with problems it might hope to tackle. Through his unification, Newton transformed rational mechanics (see 2.1), physics (see 2.2), and the relationship between them.

3. Elaborations and refinements

In this section of the paper, I remove some oversimplifications and add detail to the account of Newton's *Principia* offered in section 2, taking into account some of the secondary literature and the objections that arise therefrom.

3.1 On rational mechanics: its relationship to geometry and the epistemic status of the axioms

I have claimed that Newton's rational mechanics concerns the production of geometrical objects (especially curves) through motion, where those motions arise from any forces whatsoever. This is the sense in which rational mechanics is prior to geometry.

Garrison (1987) draws a rather different conclusion in his reading of Newton's Preface. He moves from the claim that mechanics is prior to geometry to the conclusion that (p. 612):

> For Newton, geometry had a very definite and particular content; geometry was about empirical (physical) objects extended in empirical (physical) space and constructed by God, nature, or man. … Thus for Newton, physical-empirical objects were not an interpretation of geometry but rather the interpretation of geometry; no other was imaginable.

I sympathize with Garrison's urging that we not view Newton through post-20th century formalist glasses, but nevertheless I disagree with his conclusion, in two related respects.

First, his view collapses the mathematical into the physical, making physical objects the subject-matter of geometry. However, Newton was careful to distinguish the mathematical from the physical, as I emphasized at the end of section 2, and as I discuss below (see 3.4 and 3.5).

My second disagreement with Garrison is related to the first, and concerns constructibility. Garrison suggests that the constructions of lines and circles, and of the subject-matter of geometry in general, must be carried out concretely, as a construction of physical-empirical objects. I see nothing in Newton to support this. While God is, indeed, the perfect artificer, and we may on those grounds suppose that the geometrical features of His created world are constructed exactly, rational mechanics does not require that such exact constructions have been implemented concretely rather than merely ideally in order to proceed. The very lack of concrete implementation is, after all, one element of what makes Newton's mechanics a *rational* mechanics. I take Domski's (2003) discussion of Garrison, as well as of Dear (1995) and Molland (1991), to be decisive on the issue of constructibility.

I endorse Domski's view that for Newton, "no considerations of constructilibity enter into rational mechanics" and that Newton thereby frees geometry from the constraints of practical mechanics.[53] She argues that Newton rejected Descartes's distinction between "geometrical" and "mechanical" curves, and writes:[54]

> In particular, although the origins of geometry rest on mechanical practice, for Newton, the domain of geometry is not restricted to those curves actually constructible by straightedges and compasses. The geometer does not simply start from those curves constructible by rulers and compasses and then idealize such constructions by substituting instruments with straight lines and circles. To do so would be to distinguish geometry and mechanics solely on the basis of the exactness of instruments employed in these fields, and for Newton, ''this common belief is a stupid one'' (MP, 7:289).

If this is right, then the question immediately arises as to what constraints there are on the curves of rational mechanics, the curves that are to serve as the subject-matter of geometry. In my view, Newton's criterion on admissible curves is simply that they are produced by forces and motions. This criterion becomes contentful via the definitions and the axioms, which tell us about forces, motions, and the relationships between them.

I have argued (2.3) that the justification for taking the axioms as true rests on their generality: we show their utility in solving a wide range of problems, to a high level of generality, and in so doing we justify their status as axioms. An alternative epistemology, in which a clear and evident mathematical proposition may be taken as an axiom, was present among seventeenth century French mathematicians, especially those in the Cartesian tradition[55], and it is perhaps this that Domski (2018) has in mind when suggesting that Newton took his axioms to be rationally certain. But this was not the only epistemology of axioms available at the time. For example, De Risi notes that in Germany the idea that "all axioms should ultimately be proven from the definitions" persisted, and he lists Christian Wolff's axiomless textbook as an important example.[56] Even where the axioms and postulates were judged sufficiently evident as not to require proof, "many mathematicians in the Early Modern Age believed that axioms and postulates were in fact provable from definitions"[57]. For this approach, it was the definitions that were considered the basic principles of mathematics, and the postulates and axioms were statements, derivable from the definitions, to be used in the ensuing demonstrations. It is this latter view of axioms that I think we see in Newton's *Principia*. My grounds for this claim are that I find it consistent with Newton's

[53] Domski 2003, 1123.
[54] Domski 2003, 1121-2.
[55] De Risi 2016, 599.
[56] De Risi 2016, 601.
[57] De Risi 2016, 602.

problem-oriented approach to mathematics and mechanics[58] and a fruitful way of interpreting the structure of the *Principia*.

3.2 On the distinction between axioms and laws of motion

My claim is that Newton's *Axioms, or laws of motion* serve two distinct roles, one as principles of rational mechanics and the other as principles of physics. Thusfar, I have adopted a clean terminological distinction between "axioms" and "laws" to mark these two distinct roles. This oversimplifies the situation.

First, it is easy to find counterexamples. In Book 1 of the *Principia*, Newton says things like "it is required to find the law of centripetal force"; so there are laws in rational mechanics.

More importantly, however, insisting on a clean terminological distinction risks masking the sense in which Newton's laws of motion are rightly thought of as axioms for physics, as first principles to be used in solving problems in physics. When Newton says,[59]

> as in Geometry the word Hypothesis is not taken in so large a sense as to include the Axiomes & Postulates, so in Experimental Philosophy it is not to be taken in so large a sense as to include the first Principles or Axiomes w[ch] I call the laws of motion

I think he is clearly telling us that the laws of motion have axiomatic status in experimental philosophy.

Nevertheless, the laws of motion – taken as axioms of physics – differ from axioms of rational mechanics in their epistemological status for they are to be justified inductively. Newton himself goes on to state this clearly (*ibid*):

> These Principles are deduced from Phænomena & made general by Induction: w[ch] is the highest evidence that a Proposition can have in this philosophy.

These quotations are from an exchange with Cotes which led to the addition of Rule 4 in the second edition of the *Principia*. According to Rule 4, the laws of motion – like the proportions of universal gravitation – are to be treated as "either exactly or [*quam proxime*[60]] true notwithstanding any contrary hypotheses until yet other phenomena make such propositions either more exact or liable to exceptions"[61].

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[58] Solomon 2017.
[59] Letter to Cotes, March 28, 1713, https://archive.org/details/correspondenceof00newtrich, pp. 154-5.
[60] George Smith has emphasized Newton's use of this peculiar phrase.
[61] Newton 1999, 796. One might expect Newton to provide significant inductive justification for the laws of motion, and find it puzzling that he did not. But in fact he did not take the onus to be on himself to provide such justification: in the scholia to the laws he indicated where he took such evidence to lie, and he appealed to authority as a way of evading having to make the case in detail. The point stands that in the context of physics Newton's laws are to be inductively justified.

Biener and Schliesser (2017) take a different view of the significance of the term "axiom" in *Axioms, or Laws of Motion*. They argue that Newton introduced it to indicate his increasing confidence in the truth of his hypotheses/laws, and in their applicability to a wide range of phenomena. I think that Biener and Schliesser are right that Newton became increasingly confident about the universality of his laws, as laws of physics, and I return to this below (3.6). However, I think this is not the whole story. Domski (2018) distinguishes the truth of Newton's laws from their rational certainty: the former concerns their applicability to the phenomena, whereas the latter arises from their being mathematical. In her view, Newton's laws are empirical when taken physically, but when taken mathematically they have an axiomatic status and are to be taken as rationally certain. I think Domski is right to distinguish the physical from the mathematical, and that Newton's choice of the term "axiom" tells us something about the type of justification required for the axioms/laws when considered mathematically. While Domski appeals to rational certainty, I think that it is the generality of the problems solved by means of these principles that justifies their status as axioms (see 2.3).

With all of this in mind, we arrive at a more nuanced view than that presented in section 2. The use of "Axioms, or laws of motion" as a single label for the three principles that ensue, and the placing of them outside of Books 1-3, emphasizes that these very same three principles serve as axioms for both rational mechanics and physics, and that these shared axioms turn out to be laws of motion. However, the epistemic status of axioms in rational mechanics differs from that in physics, and this distinction is crucial for understanding the philosophical structure of the *Principia*. Moreover, if we fail to recognize this difference then we may not feel the *surprise* we should on seeing that the same principles can serve as axioms for both. And so I stand by my use of "axioms" for rational mechanics and "laws of motion" for physics as a way of marking this philosophically important distinction.

3.3 On the *Principia* as a text in mechanics

In his paper "*Newton's Mathematical Principles of Natural Philosophy*: a treatise on 'mechanics'?", Gabbey presents us with a puzzle. On the one hand, the *Principia* seems to be a text in mechanics. Newton's own remarks in the Preface indicate this, and when we look back with hindsight, in the wake of later figures such as Euler and Lagrange, this seems an appropriate label[62]. On the other hand, Newton chose not to include the term "mechanics" anywhere in the title of the *Principia*, so perhaps it is not a text in mechanics afterall. Indeed, Gabbey interprets some of Newton's remarks in the *Principia* as casting doubt on the view that he thought of himself as writing a text in mechanics.

Gabbey's solution is to suggest that Newton equivocates in his use of the term, and he writes that this is understandable because the *Principia* is itself a revolutionary and transitional text.

I think my insistence that Books 1 and 2 concern rational mechanics and Book 3 physics, as Newton explicitly understood those fields of enquiry, allows us to resolve the puzzle without

[62] See Gabbey 1992, 322.

committing Newton to any equivocation. Newton's title is deliberate, for while his text incorporates mechanics (in Books 1 and 2), it is a mechanics that is targeted towards solving problems in physics, and Book 3 is a book in physics. Moreover, though I will not go through this explicitly here, the "problematic" remarks pointed to by Gabbey are also straightforwardly resolved in light of this understanding of the *Principia*.

So Newton's *Principia* is not a mechanics, but nor is it a physics: it is a *philosophical mechanics* combining a mechanics with a physics. Or so I have argued.

3.4 On Newton's three-step methodology and the "Newtonian Style"

A challenge to my reading comes from Cohen's "Newtonian Style". At first sight, I seem to be following Cohen's interpretation unproblematically, for Cohen says Books 1 and 2 are "mathematical" whereas Book 3 is a book in physics.[63] However, our interpretations of what this means diverge in significant ways.

Newton's methodology, Cohen suggests, consists of three phases, and of an iteration between the first and second phases. In the first phase, Newton treats an idealized physical system mathematically without further attention to what is "physically realistic". In the second phase, the mathematically derived results are "compared and contrasted with the data of experiment and observation"[64], and this allows a new phase one in which further deductions are made mathematically, and so on. This takes place in Books 1 and 2. The final stage of this process, phase three, takes place only when the iterative process of phases one and two reaches a level of development such that comparison with "realities of the external world" can take place[65], and this is what Newton does in Book 3:[66]

> In bk. three there is a transition from mathematical systems to the realities of the system of the world. … Then, and only then, does the question arise as to what can possibly "cause" such an "attraction".

Let's begin with Books 1 and 2. For Cohen, the subject-matter of Books 1 and 2 is idealized physical systems. That is to say, though idealized, the quantities appearing in Books 1 and 2 have a physical interpretation: the bodies are physical bodies, idealized and treated mathematically. I agree with Cohen that Books 1 and 2, though mathematical, are developed with attention to the future applicability of the results thereby obtained. However, I do not think that we should read them as treating idealized physical systems. This obscures the character of Books 1 and 2, in which Newton seeks to provide an axiomatized system of rational mechanics as a branch of mathematics treating motions and forces quite generally. As a consequence, it risks inviting a conflation of the

[63] Cohen 1980, 52ff.
[64] *Ibid*, 63.
[65] *Ibid*, 64.
[66] *Ibid*, 75.

mathematical with the physical and leads to a mistaken account of applicability, for more on which see 3.5, below.

Turning to Book 3, I agree with Cohen this is a book in physics, but I disagree with him about what this means. For Cohen, the third step in Newton's three-step methodology (see 2.2) is completed by Book 3 of the *Principia*, and does not include the consideration of causes. Cohen claims that phases 1-3 of his "Newtonian style" correspond to the three steps, with consideration of causes coming as a "sequel"[67]:

> Each of the sentences in this paragraph corresponds to one of the three successive phases of the Newtonian method in the *Principia*. … And it is only in a sequel to phase three, after the mathematical principles (established in phases one and two) have been applied to natural philosophy, that such questions as physical cause or the nature of a force need arise.

Later on, when explaining in more detail, Cohen says that phase three is "the use of the principles, laws, and rules found in phases one and two in the elaboration of the system of the world"[68], and that its "sequel" is "the process of finding a cause of gravity and of understanding how gravity may operate"[69]. He says that the move from phase two to phase three is as follows, with causes falling outside the three phases:[70]

> In phase two he found that certain forms of the basic construct (or system) led to an agreement with the phenomena to an extent that gave him confidence that the construct was not fictive … Phase three consisted of the elaboration of the system of the world, the application of the mathematical principles to natural philosophy. … In his private world, and not in the public world of the *Principia*, he then devoted himself to an exploration of the cause of the gravitating force.

In short, Cohen makes the third step – the discussion of physical causes – a "sequel" to his phase three.

The problem is that Cohen's reading does not agree with Newton's own words. According to Newton, Cohen's "sequel" *just is* the third step (see 2.2). Conceptually, Newton's three-step method seems clear. First, we have the mathematical treatment (Books 1 and 2). Second, we "come down to physics" and apply that mathematical system to the physical world, in order to identify the physical forces present in the world. Third, we seek "physical species, physical causes, and physical proportions" of those forces. This is the interpretation I advocate.

I have said that the search for causes belongs to the third step of Newton's three-step methodology, which might seem to imply a clean separation between forces (Step 2) and causes

[67] *Ibid*, 85.
[68] *Ibid*, 102.
[69] *Ibid*, 111.
[70] *Ibid*, 110.

(Step 3). This oversimplifies in two ways. First, Newton himself sometimes writes that forces *are* causes, even though in other places he seems to distinguish them.[71] Treating forces as causes makes Step 2 pertain to causes, though it challenges the dominant conceptions at the time of what is involved in giving a causal account in physics, as Janiak (2008) discusses. Second, the issue is tricky because "cause" remains untheorized in the *Principia* and "force" receives no general definition. I agree with Janiak (2008, 58) that Newton takes the term "cause" – as commonly understood at the time – to be adequate for his purposes, needing no further elaboration or clarification. In contrast to this, though we get no definition of force in general, in the *Definitions* Newton introduces technical terms for the several different force concepts that he uses in developing his arguments. So the relationship between cause and force is complicated. Nevertheless, even if forces are not causes *per se*, knowledge of physical forces is causal knowledge and so, no matter how these complexities are resolved, the important point is that Step 2 as well as Step 3 contributes to our causal knowledge. Both are needed for a complete causal account; both are properly a part of physics (see 2.2).[72]

3.5 On the mathematical, the physical, and the problem of applicability

According to Cohen's "Newtonian style", the subject-matter of Books 1 and 2 is idealized physical systems, and the applicability of the results of Books 1 and 2 to actual physical systems is achieved by the progressive removal of the idealizations from which we began. These removals takes place in phase 2 (in Books 1 and 2) and phase 3 (in Book 3). This risks a conflation of the mathematical with the physical in a way that the *Principia* is explicitly structured to avoid. Moreover, it leads to a mistaken approach concerning the applicability of Newton's mathematical results to physical systems, or so I argue.

In Books 1 and 2, Newton makes heuristic use of concrete physical systems in developing his mathematical results, and he frequently discusses such systems in scholia. Moreover, the iterative process of his method involves a complex interplay between Book 3 and the resources developed in Books 1 and 2. Newton's method, highlighted by Cohen and demonstrated in detail by George Smith and Bill Harper[73] lies at the heart of what makes Newton's methodology so powerful. All of this makes the temptation to interpret Books 1 and 2 has having physical systems – albeit idealized ones – as their subject-matter very strong. Nevertheless, it would be a mistake.

In the introduction to Book 3, Newton describes the first two books of the *Principia* as being not "philosophical" but rather "strictly mathematical", while at the same time noting that in these two books he *illustrates* the applicability of his mathematical results to actual bodies and motions in several scholia. The *Principia* developed from Newton's attempts to devise mathematical tools

[71] See, for example, Newton 1999, 382, which seems to distinguish forces from causes, and Newton 1999, 412, which seems to say that forces are causes.
[72] Newton begins this task for gravitation in the *Principia*, but does not complete it. On the incompleteness of Newton's causal account of gravitation, see Janiak 2008, ch. 3, especially p. 64, and Biener 2018, 3.

[73] See especially Smith 2002 and 2014 and Harper 2011.

for solving problems concerning the motions of physical bodies, and throughout Books 1 and 2 he provides scholia addressing the relevance of his mathematical results to the treatment of physical systems. What this shows is that Newton developed his mechanics with an eye to application. In this, I agree with Cohen[74]. Nevertheless, Books 1 and 2 do not conflate mathematics with physics, nor do they presume a physical subject-matter for a mathematical theory. Newton says that with Books 1 and 2 in place, what remains is "for us to exhibit the system of the world *from these same principles*" (emphasis added). In moving from Books 1 and 2 to Book 3, we move from mathematics (specifically, rational mechanics) to physics. It is only in Book 3 that physical systems become our subject-matter.

The position I have developed here enables me to clarify a puzzle found in Janiak's treatment of the mathematical/physical distinction in Newton's *Principia*.[75] Janiak writes that Newton's mathematical treatment of force "is not mathematical in the sense that it deals solely with mathematical entities"; rather, it deals also with physical quantities.[76] But how does this come about? One answer, to which we might be tempted, is that physical quantities are the subject-matter of Books 1 and 2. I agree with Janiak that this is not the right way to go. According to Janiak, the treatment in Book 1 is "merely mathematical" and "it is only with the physical treatment of book III in the *Principia* that we have something beyond a merely mathematical treatise on motion in general".[77] Janiak suggests that we are able to move from the mathematical treatment to the physical treatment because "the mathematical treatment in book I enables us to measure any centripetal force through a number of means, thereby enabling us to think of that force as a physical quantity"[78]. Understood mathematically, however, Book 1 sets out dependencies among mathematical quantities: it is in this sense that one quantity "measures" another. So here is the puzzle left to us by Janiak. How is it that this mathematical treatment enables us to "*think of* that force as a physical quantity" (emphasis added)? A little more is needed. The right answer, I suggest, lies in the dual nature of the *Definitions* and *Axioms, or Laws of Motion*, and in particular the dual face of the term "measure" (see 2.3, above).

I emphasize the importance of separating the mathematical from the physical because failure to do so is widespread in the literature, and leads to a variety of interpretative problems. We have already seen that Garrison (1987) collapses the distinction between the mathematical and the physical, and with what consequences (see 3.1). Guicciardini rightly draws attention to Newton's paraphrasing of Pappus in the opening lines of the Preface, in which Pappus distinguishes between the part of mechanics that is "rational" and the part that requires "manual work", but then suggests that the exactness of rational mechanics is a consequence of the subject-matter being natural and governed by mathematical laws, in contrast to the inexactness associated with imperfections of artificially constructed machines.[79] I think this further elaboration is not supported by the text of

[74] Cohen 1980, 101.
[75] Janiak 2008, ch. 3.
[76] Janiak 2008, 60.
[77] Janiak 2008, 64.
[78] Janiak 2008, 64.
[79] Guicciardini 2009, 296-7.

the Preface, in two crucial respects. First, while Newton's focus is indeed on natural forces, he makes no suggestion that the principles associated with manmade machines cannot be given an exact treatment in rational mechanics. Indeed, in a text dating from after the first edition of the *Principia*, Newton explicitly incorporates machines in his treatment[80]. Second, Newton makes no mention of mathematical laws in the Preface. This is important for my view because of the dual status that I attribute to the *Axioms, or Laws of Motion* as axioms for rational mechanics and laws of motion for physics. In my view, the exactness of rational mechanics arises directly from its formulation as an axiomatic discipline of mathematics; it does not arise from the physics, which is the domain of the laws.

Modern readers struggle to separate the rational mechanics of the *Principia* from the physics because today we think of mechanics as a branch of physics, and so when reading Book 1 of the *Principia* we see what seems to us to be physics. We read Halley's phrase "the Mathematico-Physical Treatise of the Eminent Isaac Newton" as offering a single label ("Mathematico-Physical") applied to each book of the *Principia*, rather than as a compound label applying to the treatise as a whole. Ducheyne's terminology of "physico-mathematical" for the content of Book 1 is an example of this tendency[81], and this is how I saw things myself until recently (2.3).

However, conflation of the physical with the mathematical disguises the question of how Newton's mathematical results in Books 1 and 2 are applicable to physical systems. If we follow Cohen, we read Books 1 and 2 as treating idealizations of physical systems, and by sucessive removal of these idealizations we arrive at mathematical treatments of actual physical systems. Insofar as this solves the applicability problem, it does so by assuming physical content for the mathematics throughout Books 1 and 2. In his highly interesting paper, Smeenk interprets Newton this way, situating him alongside Hobbes and Barrow, whom Smeenk says "collapsed the distinction between 'pure' geometry and physics"[82]. According to Smeenk, Newton took geometry – and indeed the mathematics of the *Principia* quite generally – "to apply directly to physical objects, whose geometrical properties are immanent in sensation rather than directly apparent."[83] If this is right, then no problem of applicability can arise: "Since the properties are not ascribed to abstract mathematical entities with a distinctive ontological status, there is no place for a worry to arise regarding how mathematical entities can stand in relation to, or represent, physical objects"[84]. However, the idealized physical systems treated in Book 1 are not themselves present in sensation either directly or immanently (think, for example, of the cases where the center of force is at rest, and where the force law differs from inverse-square), and so more must be said: the idealizations involved are theoretical, not material, and the question of their relationship to the target physical system remains. Here is one possibility: the actual trajectories of systems are exact geometrical curves (immanent in sensation) and through Newton's method we seek to ever more closely approximate them using exact theoretical trajectories. The problem with this is that the mere

80 See Gabbey 1992, 318.
81 Ducheyne 2012, 80.
82 Smeenk 2016, 310-11.
83 *Ibid*, 314.
84 *Ibid*, 314.

instantiation of an *actual* curve by a physical system is not sufficient to underwrite the iterative *counterfactual* reasoning at the heart of Newton's method (reasoning that Smeenk deftly describes), with the unwelcome consequence that Newton's method collapses into curve-fitting. Smeenk would reject this outcome, of course, but I struggle to see how he avoids it.[85]

I want to suggest a different approach. We can turn to Domski (2018) for help. As we have seen, she distinguishes between the rational certainty of Newton's laws and the truth of those laws. The former arises from their being mathematical, and the latter from their applicability to the phenomena. In her view, Newton's laws have an axiomatic status insofar as they are considered mathematical, and therefore rationally certain.[86] Insofar as they are considered physically, their justification is empirical. According to Domski (2018, p. 58), experimental evidence confirms that the mathematically certain propositions are applicable to physical objects (idealized or otherwise): it shows their truth. This is her account of applicability in the *Principia*.

I think that this is right, so far as it goes, but that it leaves us with a puzzle. It seems that every mathematically certain proposition will have to be independently verified empirically. That a given proposition is *derived* from axioms that are rationally certain and empirically confirmed tells us nothing about the empirical status of that proposition: the connections are mathematical, not physical. The generality of the axioms considered mathematically seems only accidentally related to their universality considered physically. So the applicability of the overall mathematical structure of the *Principia* remains a puzzle. One answer might be to invoke formal causation: if formal causation is both a property of mathematical demonstrations from axioms and the means by which laws govern the behaviors or bodies and forces (Biener and Schliesser, 2017), then we see how it is that the deductive structure of the rational mechanics is truth-preserving in the physics. I offer a different answer, one which lies in the dual role of the definitions and axioms/laws as principles of both rational mechanics and physics.

I insist on a rigorous separation of the mathematical from the physical: the subject-matter of Books 1 and 2 is *not* physical systems. By appreciating that Book 1 is a book in rational mechanics and Book 3 a book in physics, as I have explained the meanings of those terms, we can remove the above difficulties. While Book 1 is a *mathematical* treatise, its *interest* lies in the fact that it is derived from definitions and axioms that have a dual face, as both mathematical and physical. This is why, in so many of the scholia in Book 1, Newton discusses physical evidence and applications for his results: he is demonstrating the interest of his results for those readers whose concerns lie in physics. I have argued that Newton's *Definitions* and his *Axioms, or Laws of Motion* are independently justified within rational mechanics and within physics, and that this dual status

[85] What is the ontological status of the idealized physical systems that are the subject-matter of much of Books 1 and 2? Smeenk's view is that for Newton "'rational mechanics' offers as exact a description of material objects as the description of abstract [objects] allegedly provided by mathematics" (p. 310). But idealized physical systems are not material objects, and often the idealization and the target material system have conflicting mathematical properties; so the question of the relationship between them remains. Smeenk also suggests that the object of geometry is space, and I disagree on this point too. I think that, for Newton, geometry remains the science of magnitude. Since space is rich in geometrical properties, geometry is a powerful tool for the investigation of those properties.

[86] As we saw in 3.1, in my view it is the generality of the problems that are solved by means of these principles that justifies our taking them as axiomatic.

unifies rational mechanics and physics in a very special way. In particular, these principles form a conduit through which rational mechanics may speak about physics, and through which physics may pose problems for rational mechanics. This is because rational mechanics, as a mathematical theory, provides us with a language by which we can speak about anything that satisfies those axioms. In using the very same axioms for rational mechanics and for our laws of physics, we entitle ourselves to use the language of rational mechanics to speak about the physical world. The shared axioms yield a shared logical space for truth-preserving inferences and modal dependencies, Newton's innovative use of the latter being crucial to his method, as George Smith has emphasized. In my view, the best way to understand how the axioms are applicable is not that the laws of motion have axiomatic status in a physical theory of idealized physical bodies. Rather, these principles have two different roles, one as axioms in a rational mechanics (in which their subject-matter may be idealizations of physical systems), and the other as laws of motion in a physics (in which their subject-matter may be physical systems), and this dual character is the route through which applicability flows.[87]

3.6 On the epistemological status of the laws, as laws of physics

I have said that the epistemological status of the laws, as laws of physics, lies in their universality (see 2.3): the wider the range of phenomena to which they may be successfully applied, the greater our justification for taking them as true, within the context of physics.

Biener and Schliesser (2017) argue that Newton introduced the term "axiom" as an indicator of this universality (see 3.2). They follow the evolution of Newton's terminology in the "De motu" manuscripts from his initial use of "hypotheses" through the introduction of "laws", to the introduction of "Axioms" in Liber Primus.[88] They point out that Newton replaced "hypotheses" with "laws" at the moment that he: (a) moved from the pessimism of the "Copernican Scholium" (Smith, 2007, section 2), in which he despaired of ever determining the true motions of the planets, to optimism; and (b) recognized that his solution would use principles found Galileo's study of terrestrial gravitational motion. Looking at the second version of *Liber Primus*, they point out that the terminology of "axioms" seems to have been introduced at the same time that Newton connected his work to (c) papers on "Laws of motion" by Wren, Huygens and Wallis of the 1660s, in which they offered rules of collision, and (d) ancient mechanics (that is, the study of the five simple machines).

Biener and Schliesser suggest that Newton made these changes because he became increasingly confident that they are true of actual bodies, that they are applicable across a wide range of phenomena celestial and terrestrial, including the ancient machines and collisions, and that they are at least implicit in the mechanics of his predecessors. They claim that his concern, in

[87] As an interpretative issue concerning Newton's own position, the situation is more complex and I do not pretend to be able to argue conclusively for my approach. My claim is that my approach is consistent with Newton's statements and with his practices, and that it readily handles issues that otherwise remain unsolved.
[88] Biener and Schliesser 2017, 314.

making these terminological choices, was to signal "the truth of and broad agreement regarding his principles".

The documenting of the changing terminology over time is helpful, and I think Biener and Schliesser are right that the epistemic justification for the laws of motion, as laws in physics, lies in their universality. However, I disagree that this universality is marked by the term "axiom". Notice that (c) and (d) are topics in rational mechanics, so the introduction of the term "axiom" at this time might equally well indicate Newton's decision to attempt a unification of his results on forces and motions within a rational mechanics, as my interpretation would suggest. So I don't take the historical evolution to be decisive either way. Moreover, if I am right then the axioms require justification not just as laws of physics, but also as axioms of rational mechanics, and this demands that we show their generality (as distinct from their universality, see 3.2), which is what (c) and (d) allow. Finally, on my interpretation we gain insight from Newton's retention of both "law" and "axiom" in the label for his three principles: the term "law" indicates the universality demanded by physics and the term "axiom" indicates the generality demanded by rational mechanics.

Suppose we agree that it is the universality of the laws of motion that justifies our taking them as laws. One way to support claims of universality is through inductive evidence, and Newton's Rule 4 supports such an approach.[89] This is consistent with the view found in, for example, Harper (2011), according to which Newton's laws of motion are "empirical propositions that have already been sufficiently established to be accepted as guides to research."[90]

An alternative account of universality is offered by Friedman, who does not accept the view that Newton's laws of motion are justified inductively.[91] He emphasizes that although Newton *presented* them as familiar and accepted, this both disguises the radical innovations in Newton's deployment of these laws, and brushes under the carpet Newton's use of the third law as holding between bodies at-a-distance in the derivation of universal gravitation. In Friedman's view:[92]

> What characterizes the distinguished elements of our theories is rather their special *constitutive function*: the function of making the precise mathematical formulation and empirical application of the theories in question first possible.

For Friedman, Newtonian theory is best understood as having a tripartite structure: a mathematical part that sets up the spatiotemporal framework of the theory (Euclidean geometry); a "mechanical" part that sets up a correspondence between the mathematical part and concrete empirical phenomena (Newton's laws of motion); and a "physical or empirical" part, which attempts to formulate "precise empircal laws describing some concrete empirical phenomena" (the law of

[89] For Rule 4 see Newton 1999, 796. On the inductive justification for the laws see 2.3 and 3.2.
[90] Harper 2011, 45. In this, they differ greatly from the axioms of natural philosophy found in Rohault's physics (Chapter V). For Rohault, the axioms are "important *Truths*, which are self-evident, and which being the Foundation of all Philosophical Truths, are consequently the principal *Axioms* of Philosophy."
[91] Friedman 2001, 39.
[92] Friedman 2001, 40.

universal gravitation).[93] Of most interest for our purposes is the claim that the universality of the laws of motion arises from their constitutive function.

Friedman's overall partition of the *Principia* differs significantly from that presented here. Geometry, as I read it in the context of the *Principia*, is a science of magnitude primarily, and thereby of space (among other things), and the "mathematical part" is the rational mechanics of Books 1 and 2, not just the spatiotemporal framework of the theory. I agree with Friedman that Newton's laws of motion are critical for the applicability of the mathematical to the physical, but he and I differ in how this is achieved. We agree that Newton's treatment of universal gravitation in Book 3 is distinct from his treatment of the laws of motion, and for me this is because it takes place in the physics. Friedman's account partitions and conceptualizes the *Principia* very differently from my approach, but I agree with Friedman that there is a constitutive role for the axioms/laws. I think this holds for both the rational mechanics and the physics, and plays out differently in each. As a result, I would like to return to Friedman's account of the constitutive role of the axioms/laws in light of the dual role of the axioms/laws, but that is a task for another paper.

My point here has been that the justification for taking Newton's laws of motion as axioms for physics lies in their universality, and that this may be justified either inductively or through appeal to their constitutive role.

4. Conclusions

Stan and I use the label "philosophical mechanics" for projects that seek to combine rational mechanics with physics into a single account of a given range of phenomena. I have argued that Newton's *Principia* is powerfully understood as just such a project. Approached in this way, we can unite into a coherent whole Newton's description of rational mechanics in the Preface, his claims that Books 1 and 2 are mathematical, his claim that in Book 3 we move from the mathematical to the physical, and his three-step methodology in which we begin with mathematics and then, moving on to physics, we first identify which forces obtain in the world and then consider the causes of those forces. The unification is achieved via the *Definitions* and the *Axioms, or Laws of Motion*, which serve as principles in both the rational mechanics and the physics. The interpretation and justification of these principles has a correspondingly dual aspect.

This interpretation of Newton's *Principia* offers a new angle on existing questions in the secondary literature, including the sense in which Books 1 and 2 are to be understood as "mathematical"; whether or not the *Principia* is a text in mechanics; why Newton came to adopt the dual label "Axioms, or laws of motion"; the epistemic status of the axioms; the relationship between the axioms and the Definitions; in what sense Book 3 is incomplete as a physics; and the problem of applicability (how it is that the mathematics of Newton's *Principia* is applicable to the physical world).

Equally important, for my purposes, is that it provides us with an example of a philosophical mechanics that was highly visible, powerful, and influential throughout the 18th century. On the

[93] Friedman 2001, 80.

one hand, it set standards for what a successful integration of rational mechanics with physics should look like. On the other hand, Newton's *Principia* offered an *incomplete* physics, and 18th century attempts and failures to complete the physics provide us with insights into the limitations of philosophical mechanics as a project for understanding the natural world. In our book, Stan and I follow parallel attempts to provide a philosophical mechanics of collisions. The rules of collision, dating back to Wren, Huygens and Wallis, lie at the heart of any rational mechanics of collisions. A successful philosophical mechanics of collisions would unite such a rational mechanics with a causal story of the collision process by which bodies undergo changes in their state of motion due to contact action.

The lens of philosophical mechanics allows for a comparison of the two options for body-body action that dominated natural philosophy at the time: contact action, as exemplified by collisions, and action-at-a-distance, as exemplified by Newtonian gravitation. The former led to a progressive research program, as the work of George Smith has done so much to demonstrate.[94] The latter did not. For philosophers of the early 18th century, this outcome was not in sight. Indeed, it was Newtonian attraction, or action-at-a-distance, rather than action-by-contact, that was controversial and in doubt. By the end of the century, the positions were reversed. The successes and failures of each turned out to have long-lasting consequences for philosophy and physics. But that is a story for another day.

To conclude, then. This paper concerns a rather abstract conception of the *Principia*. As such, it is far removed from the details of Newton's work, and the painstaking interplay between empirical data and mathematical reasoning that Smith emphasizes and so masterfully conveys in his research and teaching. My original plan had been to compare the evidential situation in the eighteenth century for gravitational physics/mechanics with that for non-gravitational physics/mechanics. This is more in keeping with Smith's approach, but I never got that far. My comparison was to be carried out within the framework of philosophical mechanics, and the "introductory" section on Newton's *Principia* as a text in philosophical mechanics grew to become the entire paper. Nevertheless, there is a common theme between this paper and Smith's work. In order to appreciate Newton's achievements, we have to go back to the areas of research in which he worked, as they stood prior to the *Principia*, to see the problem-space as it presented itself to people at the time. When we do this, we see in detail the extraordinary ways in which Newton went beyond what anyone else was able to do, transforming entire fields of research in the process. No-one has done more to demonstrate the value of this methodology for Newton scholarship than George Smith.

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[94] See especially Smith 2014.

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