

quired by being transported with it, might well be hurled forward, if they were not tied down.

WHY THE ROLLING OF A SHIP CAUSES VOMITING.

It is for the same reason that the sea causes the rolling of a ship, and even more the turmoil of a storm, makes men sick and makes them vomit, especially if they are not accustomed to the sea. For, the liquids in their bodies only gradually gain a movement in *harmony* with that of the ship, and until they have acquired it, there is disorder and commotion in the body, which takes the form of vomiting and other illnesses; and so, almost the same thing happens in the bodies of men as in a vase filled with water that is spun around; for the water only slowly acquires the motion of the vase, and it maintains it some time after this motion is stopped.

CHAPTER TWENTY-ONE

*Of the force of bodies*

[In §§.557–72 Du Châtelet describes the ways in which force acts on bodies; for example, force is successively acquired. Force acts even if just as a tendency when a body resists; for example, a body resists the force of gravity when placed on a table. This "harmless effect" of the force is *force morte* and is retained by the body as long as its motion is opposed by an invincible obstacle. The formula for determining *force morte* is  $mv$  (mass  $\times$  velocity). She explains that all mathematicians agree on this definition and this formula for its determination. Leibniz was the first to distinguish between *force morte* and *force vive* in his memoirs for the *Acta Eruditorum* (1686 and after). As she describes it, *force vive* is the successive acquisition of force by a body. She uses gravity as an example and cites Galileo's formula measuring the force of gravity as the square of the speed of fall. Thus, she concludes that *force vive* is measured by the square of the speed of motion of the body multiplied by its mass,  $mv^2$  (expressed as  $\frac{1}{2}mv^2$  today). Although experiments confirm this conclusion, she notes that it is considered a "kind of heresy in physics." She then answers the principal objection to *force vive*, the argument of "time," as a determining factor in the measurement of force. Opponents, she explains, argue that a force increases as it takes longer to act on a body; for example, a spring closing. She responds that the force can only be measured by the obstacles it overcomes and by which it is consumed; the time is of no consequence, and making time a determining factor in the equation

for force leads to "absurdities," such as perpetual mechanical motion, continual motion in infinite time, which all agree is impossible.]

SOME REFUSE TO ACCEPT *FORCES VIVES* WHILE ACKNOWLEDGING THE EXPERIMENTS THAT ESTABLISH THEM.

§.573. *Forces vives*<sup>109</sup> may be the only point of physics which some still dispute while acknowledging the experiments that prove it; for if you ask those who reject them what would be the effects of two bodies equal in mass on two equal obstacles, but the speeds of which are 4 and 3, they will answer that one will be an effect, as 16 and the other as 9. Now, it is easy to see that, whatever distinction and whatever modification they next bring to this acknowledgment that the force of truth draws from them, it always remains certain that the effect being squared, there must have been a squared force to produce it.

§.574. It would be pointless to report to you here all the experiments that prove this truth, you will one day see them in the excellent memoir that M. Bernoulli presented to the Academy of Sciences in 1724 and in 1726, and found in the *Recueil des pièces* [Collection of Memoirs], which won, or merited the prizes it awards. And you have already seen a part of it in the memoir that M. de Mairan gave in 1728 to the Academy against *forces vives* that we read together, and in which the famous proceeding is explained with much clarity and eloquence.

EXAMINATION OF SOME PARTS OF M. DE MAIRAN'S MEMOIR AGAINST *FORCES VIVES*.

As this work appears to me to be the most ingenious that has been produced against *forces vives*, I will pause to take the time to remind you here of some passages, and to refute them.

M. de Mairan says, numbers 38 and 40 of his memoir: "That the force of bodies should not be measured by the spaces traversed by the moving body in the slowed motion, nor by the obstacles overcome, springs closed, etc. but by the spaces not traversed, by the parts of matter not displaced, the springs not closed, or not flattened: now," he says, "these spaces, parts of matter, and springs are like simple speed. Thus, etc."

One of the examples he gives is that of a body that goes back up to the same height from which it fell with the force acquired in falling, and that in going back up overcomes the obstacles of gravity: "For a body fallen from

109. Du Châtelet, like her contemporaries, refers to *force vive* in the singular and the plural, *forces vives*, but it is the same phenomenon.

a height 4 and which acquired 2 of speed in falling, would in going back up by a uniform motion, and with this speed 2, travel a space 4 in the first second; but gravity which pulls it down, making it lose in this first second 1 of force and 1 of speed, it only traverses 3 in the first second, the same as in the second second where it still has 1 of speed and 1 of force; whereas it would traverse 2 in a uniform motion, it only traverses one, because gravity makes it lose *one*. What are the losses of this body, *one*, in the first second, and *one* in the second? This body that had 1 of speed, has lost 2 of force, so the forces were as its speeds," concluded M. de Mairan, "not as the square of its speeds."

But to see the fault in this reasoning, it suffices to consider (as in §.567)<sup>110</sup> the action of gravity as an infinite sequence of equal springs, which communicate their force to falling bodies, and which the body contains in rising again; for, then, it will be seen that the losses of a body that rises are as the number of closed springs, that is to say, as the spaces traversed, not as the spaces not traversed.

In the obstacles overcome, as with the displacements of matter, the closed springs, etc. even by way of hypothesis or supposition, it is impossible to reduce slowed motion to uniform motion, as M. de Mairan advances in his memoir, and whatever esteem I have for this philosopher, I dare insist that when he says in numbers 40, 41, and 42, *That a body, which by a slowed motion, closes three springs in the first second, and 1 in the second, would close 4 in this first second, and 2 in the second by a uniform motion and a constant force*, he is saying, I am not afraid to venture this, an entirely impossible thing. For it is as impossible for a body with the force necessary to close 4 springs to close 6 (whatever supposition is made), as it is impossible that 2 and 2 make 6. For if one supposes with M. de Mairan that the body has not consumed any part of its force to close 4 springs in the first second of a uniform motion, I say that these 4 springs would not be closed, or that they would be so by some other agency. If one supposes the contrary, that, having exhausted a part of its force to close these first three springs in the first second, and having only the necessary force to make it close a spring in the second second, the body would take back a part of its force to close two in this second second by a uniform motion (for one or the other of these suppositions must be made), one obviously supposes in this last case that the body has renewed its force, which is beside the question. Thus, it is not true that the total force of a body is represented by what it would have done if it had not been consumed; for it could

110. In §.567 Du Châtelet describes the accumulation of force as a body ceded to successive pressures exerted on it and thus acquired by it.

never make an effect greater than that which destroyed it, and it only potentially contained what it deployed in the effect produced. Thus, this very subtle reasoning, which initially might seem alluring, relies only on this false principle, that the quantity of motion and the quantity of the force are the same thing, and that the force can be supposed to be uniform like the motion, although it has overcome part of the obstacle that must consume it. But that is entirely false, and cannot be accepted even as supposition. For to suppose simultaneously that a force stays the same, and meanwhile produces part of the effects which must consume it, that is to suppose contradictory things. Thus the measure of the force in slowed motions is *not the parts of undisplaced matter, the springs not pulled, the spaces not traversed in going up, but, the spaces crossed in rising, the parts of displaced matter, and pulled springs.*

M. de Mairan goes on to say in number 33 that "just as a force is not infinite, because the uniform motion it produced in an unresisting space would never cease, it does not strictly follow either that the motor force of this same body is bigger because it lasts longer." But it is easy to see that in uniform motion supposed eternal, there is no destruction of force, whereas when the motor force during a doubled time has disturbed squared obstacles, there has been a real expenditure of force, which cannot have happened without a base of force squared, and that, thus, the two cases cannot be compared.

I flatter myself that M. de Mairan will consider the remarks I have just made on his memoir as proof of the regard in which I hold this work. I confess that he has said all that could be said in favor of a bad cause; thus, the more seductive his reasoning, the more I felt obliged to make you see that the doctrine of *forces vives* is not undermined by it.

VERY OBVIOUS REASONING WHICH PROVES FORCES VIVES.

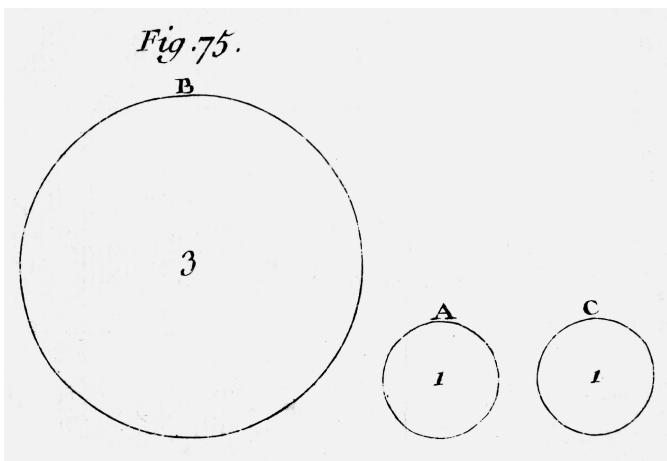
§.575. This doctrine can be confirmed by a very simple argument, which everyone makes naturally when the occasion arises: if two travelers walk equally fast, and one walks for one hour, and makes one *lieue*, and the other two *lieues* in two hours, everyone acknowledges that the second made double the distance of the first, and that the force he used to cover two *lieues* is double that which the first used to walk one *lieue*. Now, supposing that a third traveler covers these two *lieues* in one hour, that is to say, that he walks at double the speed, it is evident again that the third traveler, who makes two *lieues* in one hour, uses two times the force used by the one who walked these two *lieues* in two hours. For we know that the faster a courier must walk to cover the same distance in less time, the more force he needs, which all couriers understand so well that they all want to be better paid the faster

they go. Now, since the third traveler uses two times more force than the second, and the second uses two times more than the first, it is obvious that the traveler who walks at double the speed during the same time, uses four times more; and consequently the forces that these travelers expended will be as the square of their speeds.

§.576. The enemies of *forces vives* manage to discount most of the experiments that prove them, because they cannot deny them. They reject, for example, all those done showing the impression bodies make in soft materials, and it is true that there is inevitably always confusion in the results of these experiments, and in the examples one deduces from animate creatures, strange circumstances which prolong the disputes.<sup>111</sup>

ACADEMY OF PETERSBURG, FIRST VOLUME. DECISIVE EXPERIMENT OF M. HERMANN IN FAVOR OF FORCES VIVES.

§.577. But M. Hermann reports a case that leaves no place for any subterfuge, and in which it cannot be disputed that the force of a body was squared by virtue of a doubled speed.<sup>112</sup> This is the case in which, for example, a ball A which has 1 of mass, 2 of speed, successively hits on a horizontal plane, supposed to be perfectly smooth, a ball B at rest, which has 3 of mass, and a ball C that has 1 of mass; for this body A will give a degree of speed to ball B whose mass is 3, and it will give the remaining degree of speed to ball C, which it



111. This is a description of 'Gravesande's experiments from his *Physices elementa mathematica* [Mathematical Elements of Physics] (1720).

112. Jacob Hermann (1678–1733), a mathematician, was known for his work in mechanics. Du Châtelet is referring to a memoir he wrote while at the Academy of St. Petersburg.

next encounters, and whose mass is 1, that is to say, equal to its own; and this body A, having then lost all its speed, will stay at rest.

Now let us examine what the force will be of bodies B and C to which body A communicated all its force and all its speed; certainly the mass of body B being 3 and its speed 1, its force will be 3 even in the opinion of those who refuse to accept *forces vives*; body C, whose speed is 1 and mass 1 will also have 1 of force: thus body A will have communicated the force of 3 to body B and the force of 1 to body C. Thus body A with 2 of speed gave 4 of force. This means that it had this force; for, if it had not had it, it could not have given it; thus, the force of body A, which had 2 of speed and 1 of mass, was 4, that is to say, as the square of this speed multiplied by its mass.

[Fig. 75 appeared here in the original.]

§.578. There is an admirable correspondence between the way body A loses its force by the impact in this experiment, and the way a body that rises up again by the force acquired falling, loses its own because of the redoubled pull of gravity. For a body that, with a speed of 2, will rise up to a height 4, loses 1 of speed when it has risen up again to a height 3, just as ball A loses 1 of speed in setting ball B in motion, whose mass is 3; and the body that rises up again loses the second degree of speed that remains to it, in rising from a height of 3 to a height of 4, that is to say, in traversing a space one-third of the first, just as body A loses the degree of speed left to it in hitting body C, one-third of body B. Thus the same thing happens, either because the force of bodies is communicated to them by impulsion or as an effect of their gravity.

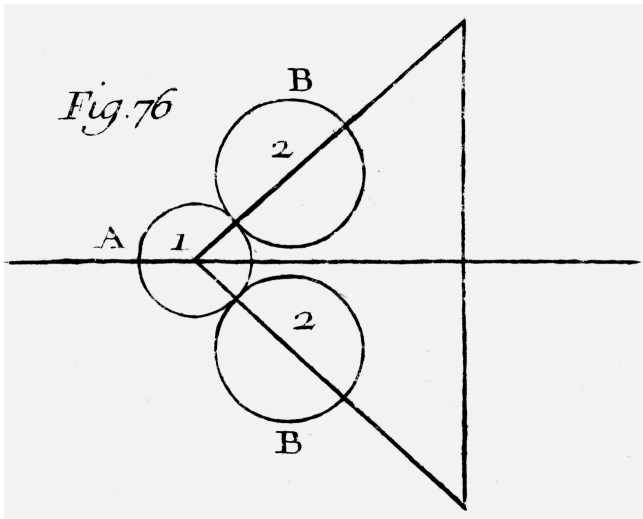
HOWEVER, THE DIFFICULTY WITH TIME ALWAYS REMAINS IN THIS EXPERIMENT.

§.579. Although in this experiment of M. Hermann's, a body with 2 of speed communicated 4 degrees of force to bodies equal to it, which can then exert this force and communicate it to other bodies, which leaves no place for pretexts that one alleged against most of the other experiments which prove *forces vives*. However, the difficulty with time (if it is one) always remains in this experiment, since the ball A only communicated its force to balls B and C successively. Thus all the adversaries of *forces vives*—M. Papin who rejected them and M. Leibniz, their inventor,<sup>113</sup> and M. Jurin, who recently declared against this opinion—have always challenged M. Leibniz and the partisans of *forces vives*, to demonstrate to them a case in which a doubled speed produced

113. *Inventor* in the sense of first conceptualizing them as distinct.

a squared effect in the same time, in which a simple speed produces a simple effect, going so far as to promise to accept *forces vives*, if such a case could be found in nature. This is how M. Jurin puts it: *Id si facere dignati fuerint me ipsis discipulum, parum id quidem est, at multos egregios viros ausim promittere.*<sup>\*114</sup>

§.580. As the laws of motion do not permit, when a body hits a single other one, for it to transmit all its force to another with four times the mass by a single hit, M. Leibniz, in order to meet this kind of challenge, resorted to a lever, by means of which he succeeded in transmitting by a single hit all the force of a body to another with four times the mass, to which it communicated half of its speed. But the fact of the lever gave rise to exceptions that made M. Leibniz's experiment unproductive for the purpose of converting his adversaries. Thus the objection based on the difficulty with time always remained.

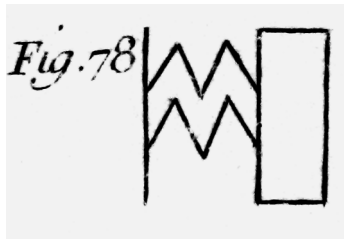
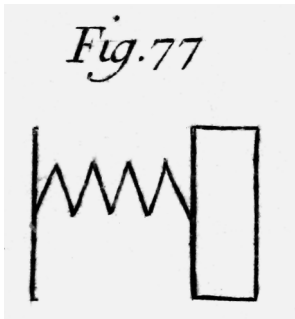


EXPERIMENT THAT ENTIRELY DESTROYS THE OBJECTION BASED ON TIME.

§.581. But this objection was completely overturned by finding the case the adversaries of *forces vives* believed could not be found. This is the case in which a body A freely suspended in the air whose speed is 2, and the mass supposed as 1, at the same time hits at an angle of 60 degrees two bodies B

114. Du Châtelet here makes her own note to give the translation of Jurin's quotation: <sup>114</sup>\*And if they can find such an effect in nature, I promise them, not only to be their disciple, which

and B, the mass of each of which is 2, for, in this case the body striking A stays at rest after the hit, and the bodies B and B divide its speed between them, each moves by a degree of speed. Now these bodies B and B, whose mass is 2 and who have each received a degree of speed, have each acquired 2 of force, whichever way one looks at it. Thus body A with a speed of 2 communicated a force of 4 at one and the same time. This is precisely the case required by the adversaries of *forces vives*; thus, this experiment makes the objection based on the difficulty of time, about which up to the present the enemies of *forces vives* have made such a fuss, collapse entirely.



OTHER PROOF DRAWN FROM THE TIME IN WHICH SPRINGS COMMUNICATE THEIR FORCE.

§.582. In addition, the force is always the same, whether it has been communicated in a short time or a long time. The time in which springs communicate their force, for example, depends on the circumstances in which they are deployed; for there are circumstances in which the force of

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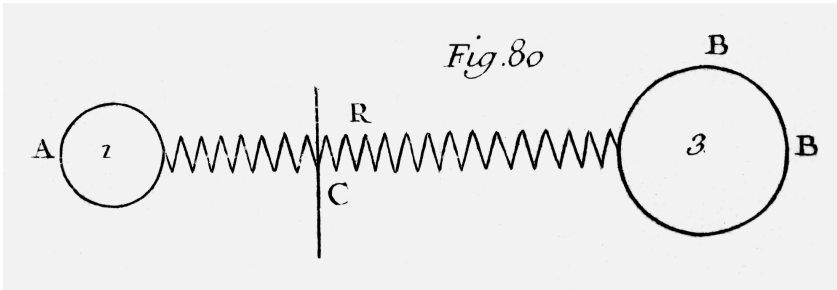
would mean much; but to find more distinguished ones for them." Denis Papin (1647–1712?), though French, worked with Huygens and became a professor of mathematics at the University of Marburg. James Jurin (1684?–1750), the English mathematician and well-known supporter of Newton, later corresponded with Du Châtelet on this particular point in her *Foundations*.



a spring can be transmitted in the same body faster than in other circumstances. Yet the force that this spring communicates is always the same. Thus, four equal springs will communicate the same force to the same body, whether they communicate it in one, two, or three minutes, as in Figs. 77, 78, and 79, and this time could be infinitely varied, depending on whether these springs were more or less at liberty to act, though the force communicated was always the same; thus, the time is immaterial to the communication of motion.

ANOTHER OBJECTION TO FORCES VIVES.

§.583. There is yet another objection to *forces vives*, which at first appears fairly strong; it arises from what happens when two bodies hit each other with speeds that are in inverse proportion to their mass, for if these bodies are without perceptible spring they will stay at rest after the collision. At first it would seem as if the body, which has the most speed having the most force, according to the doctrine of *forces vives*, must push the other body before it.



RESPONSE.

MACLAURIN PRIZE ACADEMY PIECES. BERNOULLI PRIZE PIECES.  
DISCOURSE ON MOTION.<sup>115</sup>

But to understand how two bodies with unequal force can, nonetheless, stay at rest after the collision, let us consider a spring R, which releases its tension at the same time on both ends, and which pushes at either end bodies of unequal mass, the inertia of these bodies being the only obstacle they oppose

115. Colin Maclaurin (1698–1746), a Scots mathematician and professor at Edinburgh University, was active in the prize competitions of Europe's Academies.

to the release of the tension of the spring; and this inertia being proportional to their mass, the speeds the spring will communicate to these bodies will be in inverse proportion to their mass; and consequently they will have equal quantities of motion, but their forces will not be equal, as M. Jurin and some others would infer. These forces will be to each other as the length CB to length CA, that is to say, as the number of springs that acted on them; thus, their forces will be unequal and will be to each other as the square of the speed of these bodies multiplied by their mass.

Now when the spring R is released up to a certain point, if these bodies returned toward it with the speeds it communicated to them by releasing, it is easy to see that each of these bodies would have precisely the necessary force to return the parts of the spring that acted against it in their first state of compression, and that they would use unequal force to close this spring, since in releasing, it had communicated to them unequal forces, which they consumed in closing it; and if the spring was stopped in its state of compression when these bodies have just closed it, the two bodies, all of whose forces had been used to close it, would then remain at rest.

Now, when two bodies that are not elastic collide with speeds that are in inverse proportion to their masses, they have on each other the same effect as one has just seen, the effect that body A and body B had on the parts of spring R in order to close it, and it is easy to see by this example how bodies can consume unequal forces in the giving way of their parts and stay at rest after the collision.

#### EXPERIMENT THAT CONFIRMS THIS ANSWER.

§.584. M. 'sGravesande created an experiment that wonderfully confirms this theory. He took a firm ball of clay and, using Mariotte's Machine,<sup>116</sup> he made it collide successively with a copper ball, whose mass was 3 and speed 1, and with another ball of the same metal whose speed was 3 and mass 1, and it happened that the impression made by ball one, whose speed was 3, was always much greater than that made by ball three with the speed of 1, which indicates the inequality of the forces. But when these two balls with the same speeds as before collided at the same time with the clay ball freely suspended from a thread, then the clay ball was not set in motion

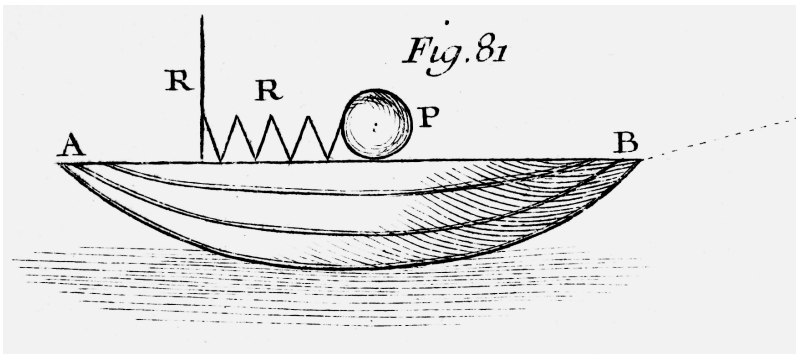
116. This is a second experiment by 'sGravesande to support the concept of *forces vives*. Mariotte's Machine was a simple structure with balls of different materials hung so that collisions could be enacted and observed. As modified by Musschenbroek, the Dutch experimentalist and instrument maker, the collisions on the machine happened against a board with markings that could be used to measure the recoil of the balls.

and the two copper balls stayed at rest and equally depressed the clay; and these equal impressions having been measured, they were found to be much greater than the impression that ball three with the speed of 1 had made when it only hit the firmed clay ball, and less than that which had been made by ball one with the speed 3. For ball three had used its force to make an impression on the clay ball, and its impression having been augmented by the effort of ball one that pressed the clay ball against ball three, diminished the impression of this ball one. Thus soft bodies that collide with speeds in inverse proportion to their masses, stay at rest after the collision, because they use all their force to mutually impress their parts. For it is not simple rest that holds these parts together, but a real force, and in order to flatten a body and drive into its parts, this force, named *coherence*, must be overcome, and in the collision the force used to drive into and impress these parts is consumed.

M. JURIN'S REASONING AGAINST *FORCES VIVES*.

§.585. The most specious reasoning made against *forces vives* is that of M. Jurin, reported in the *Philosophical Transactions*. . . .<sup>117</sup>

[Du Châtelet describes his assertion that the force of a body on a moving plane "will be its simple speed multiplied by its mass, and not as the square of this speed," and then proceeds to answer it.]



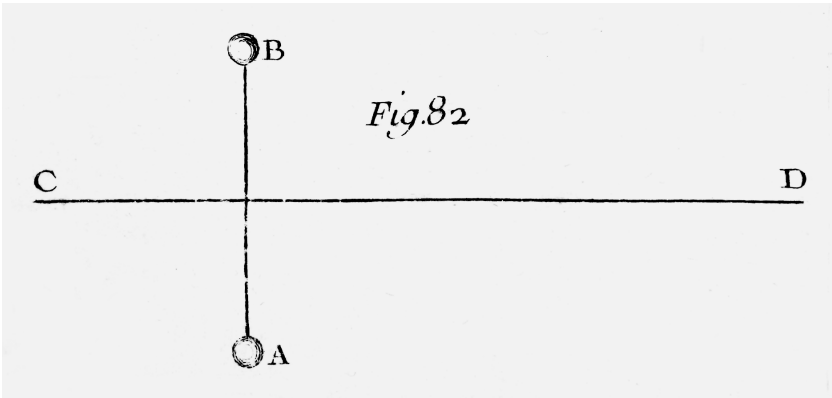
WHERE THE FLAW IN THIS REASONING LIES.

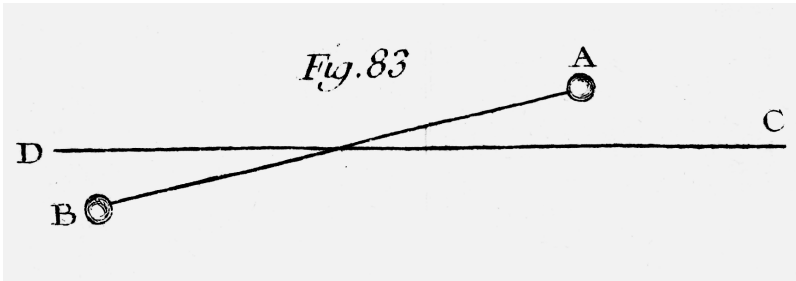
Here is where the flaw in this reasoning lies. Let us suppose, instead of the moving plane of M. Jurin, a boat, AB moving on a river in the direction BC

117. The *Philosophical Transactions* was the publication of the Royal Society of London, the English equivalent to the Royal Academy of Sciences in Paris.

with the speed 1, and the body P carried on the boat. This body acquires the same speed as the boat; thus its speed is 1. If a spring is attached in this boat, capable of giving to body P a degree of speed, this spring, which communicated to body P the speed of 1 off the boat, will not communicate it any more when it is carried on the boat; for the rest, against which the spring presses in the boat, not being immovable rest, and the boat yielding to the effort that the spring makes toward A, this spring releases at the same time from both ends, and this reaction must be taken into account. Thus, the spring will not communicate to body P the speed 1 in the boat, but it will communicate this speed less something, and this difference will be more or less great, according to the proportion that exists between the mass of boat AB and that of body P and the same quantity of *force vive*, which was in the boat AB in the spring R and in the body P before spring R was released, will exist after its release in the boat and in the body taken together. Thus, this case that M. Jurin defies all philosophers to reconcile with the doctrine of *forces vives* is only founded on this false supposition that the spring R will communicate to body P, carried on a moving plane or in a boat, the same force that it would communicate to it if the spring were pressing against an immoveable obstacle and at rest, but this is not the case, and cannot be, except in the case when the mass of the boat is infinite in relation to that of the body.

M. NEWTON MADE THE FORCE OF BODIES PROPORTIONAL TO THE QUANTITY OF THEIR MOTION.





INEXPLICABLE PHENOMENON WITHOUT THE DOCTRINE OF *FORCES VIVES*, WHICH LED M. NEWTON TO CONCLUDE THAT THE TOTAL FORCE IN THE UNIVERSE WAS VARIABLE.

§.586. Although authority must be counted when truth is at issue, I feel obliged to tell you that M. Newton did not acknowledge *forces vives*, for the name of M. Newton is in itself nearly an objection. In the last question of his *Opticks* this philosopher examines the movement of an inflexible stick AB, at both ends of which have been attached bodies A and B, and he supposes that the center of gravity of this stick AB that he only considers as a line, moves the length of the straight line CD, while the bodies A and B turn continuously around this center, when the line AB is perpendicular to CD (as in figure 82) the speed of body A is zero, and that of body B is 2. Thus the motion of these bodies is then 2; but when this line AB is coincident or almost coincident with line CD (as in figure 83) then the sum of the motions of bodies A and B becomes 4. M. Newton concludes from this consideration and that of the inertia of matter that motion is constantly diminishing in the universe; and lastly that our system will some day need to be formed anew by its Author, and this conclusion was a necessary consequence of the inertia of matter, and the opinion held by M. Newton that the quantity of force was equal to the quantity of motion.<sup>118</sup> But when the product of the mass by the square of the speed is taken as force, it is easy to prove that the *forces vives* always remain the same, although the quantity of motion varies perhaps at each instant in the universe, and in all the cases, and especially in that which I have just cited from M. Newton, the *forces vives* stay invariable; whatever the position of the line AB in relation to line CD described by its center of

118. Du Châtelet is referring to Query 31 of the *Opticks* in which Newton considers the nature and behavior of particles. From the description of the motion of *globes* on the balance, he concludes that motion, or force, is lost. The consequence of this would be a need for divine intervention to replenish the force in the universe. See Isaac Newton, *Opticks* (New York, 1979) [1730 ed.], 397–98.

gravity. Thus, the continual miracles, which result from the position of this line AB have no place in the doctrine of *forces vives*.

[In §§.587–90 Du Châtelet concludes by presenting metaphysical arguments in favor of *forces vives* drawn from Descartes and given mathematical expression by Leibniz. She explains that this “conservation of an equal quantity of force” in the universe in all kinds of collisions negates the need for “miracles,” for divine intervention, and is worthier “of the grandeur of the wisdom of the Author of nature.”]