Emilie Du Châtelet, *Foundations of Physics*, 1740.
Translated by Katherine Brading *et al*.[1] at the University of Notre Dame and Duke University. Footnotes are ours except where otherwise indicated.
Du Châtelet's marginal notes are placed in **{bold}** in the closest appropriate place in the text. Please see the French original for the position of each note in the margin alongside the paragraph. Figures are available in the original text, and online via the BNF.

Chapter 18. Of the Oscillation of Pendulums

443. {What a pendulum is.} A pendulum is a heavy body, suspended by a thread, and attached to a fixed point around which it can move by the action of gravity, once it has been set in motion.

444. {What is the cause of its vibrations.} If body P **{Fig. 56}**, suspended by thread BP, is attached to immobile point B, and being pulled from position BP, perpendicular to the horizon, is raised to C (for example) and then left to itself, then it is certain that by the force of its gravity it will descend towards the earth as far as it can.

If this body were completely free, it would follow the perpendicular line CL. But, being attached at B by the thread BP, it can only partly obey the effort of gravity that carries it along this line CL. Thus, it is constrained to descend along arc CP.

Body P, in falling from C to P along arc CP, acquires the same speed as if it had fallen from perpendicular height EP, and as a result it has the necessary speed to return to this same height, along the same curve in equal time, supposing that some cause changes its direction without altering its speed (§319, 1.). This cause, which changes the direction that gravity impresses on body P, is thread BP; for once the body has arrived at P, it cannot descend further toward the earth; nevertheless it conserves all the speed that gravity had impressed upon it from C to P. Now, if in this moment gravity ceased to act upon the body, and if it were no longer restrained by the thread BP, it would follow the straight line PD, tangent to circle CP in which the body is moving (first law, §229). But since the thread BP opposes the body's gravity with an invincible obstacle at point P, although the body has a tendency to escape along the tangent PD the thread BP draws it back at the first moment to make it begin another tangent, from which it is at every moment drawn back. Thus, the thread BP, changing the direction of this body at every moment, makes it traverse the arc of circle PR. This arc PR is equal to CP, for by the force acquired in falling from C to P this body must return to the same height from which it had fallen, since gravity removes along PR all the force it had given to the body along CP (§318).

It is in almost the same way that celestial bodies make their revolution in curves around the

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Sun without falling into this star, as I will explain when talking about Astronomy.

Once body P has arrived at B, all the force that it had for reascending having been consumed, it will due to its heaviness fall back to P, from where it will reascend to C, and so on **{Fig. 56}**. **{What a vibration is.}** This to and fro of Pendulum BP from C to P and from P to R is what we call the oscillations, the vibrations of this Pendulum, of which we see that heaviness is the unique cause.

445. {Pendulums in their vibrations describe arcs of a circle.} Because body P is restrained by thread BP along the circumference of the circle GPM, of which thread BP is the radius, the arc CPR that the body will describe will be an arc of a circle.

446. Thus, thread BP, to which the oscillating body is attached, is for this body an obstacle that opposes the force that carries it toward the earth, and it is this force of gravity alone that makes this body undergo vibrations.

447. {Definitions.} {Fig. 56} The straight line SBT parallel to the horizon, and passing through point B around which Pendulum BP oscillates, is called the axis of oscillation, and point B, to which thread BP is attached, is called the point of suspension.

In Pendulums, we consider the weight of the suspended body as being concentrated at a single point.

448. Pendulums can be simple or compound.

449. {Of Simple Pendulums.} Simple Pendulums are those to which only one weight is suspended. **{Of Compound Pendulums.}** Compound Pendulums are those to which several weights are attached at different distances from the point of suspension.

450. {A Pendulum would oscillate for all eternity in a non-resisting medium without friction.} {Fig. 56} If air did not resist the motion of the Pendulum, and if the thread did not experience any friction at its point of suspension, we easily recognize[2] that a body having begun to oscillate from C to P and from P to R would continue these oscillations for all eternity; since in falling from C to P it acquires the speed necessary to reascend from P to R, and having arrived at R it falls back to P by the force of its gravity, to then reascend to C by the force acquired in descending, and so on.

451. But as we do not know of any body that is exempt from friction, and since the air in which Pendulums oscillate resists their movement, every Pendulum left to itself in the end loses its

[2] The French reads: "on sent aisément".

motion, and at the end of a certain period of time the arcs that it describes diminish until finally, the arcs becoming infinitely small, the Pendulum stays at rest in the direction perpendicular to the horizon that is its natural direction.

452. However, we abstract air resistance and the friction the Pendulum experiences at its point of suspension when we treat the oscillations of Pendulums, because we consider them in a very short time only, and in a small space of time these two obstacles have no perceptible effect on the Pendulum.

453. {Fig. 57} If the arcs CP and PG that body P traverses in its vibrations are very small, they will differ very little in length and in inclination from the chords MP and RP that subtend them. Thus the body will make a half oscillation from C to P in a time perceptibly equal to that which it would take to traverse chord MP or the diameter AP of circle ACP in which it oscillates (§433).

454. {Oscillations of very small but unequal arcs are made in perceptibly equal times.} It follows from this that a Pendulum making its oscillations in very small arcs makes them in perceptibly equal times, even when the arcs that it traverses are not equal. For these arcs having been traversed in times perceptibly equal to those that the body would take to traverse the chords that subtend them, and these chords all being traversed in equal time (§433), Pendulum P will traverse the small arcs CPG, DPF in perceptibly equal times **{Fig. 57}**. Thus, for two Pendulums of equal length oscillating along different small arcs of a circle, their vibrations are so equal that in a hundred vibrations they differ by hardly a single one.

455. {The speeds acquired along unequal arcs are as their subtended chords.} {Fig. 58} The speeds of bodies that oscillate in the different arcs CB and DB of a circle are to one another, once they have arrived at point B, as the subtended chords of the arc that they traversed. For if we take the horizontal lines CF and DE, we see that the speeds that the body acquired in falling along arcs CB and DB are the same as those that it would have acquired in falling perpendicularly from F to B and from E to B (§438). Now the speed acquired from F to B is to the speed acquired from G to B as the square roots of GB to FB (§315, number 4) or as line CB is to line GB (§429). In the same way the speed acquired from E to B is to the speed from G to B as the square roots of EB to GB, that is to say, as line DB is to line GB, and as a result the speed from F to B is to that from E to B as chord CB is to chord DB. But the speed acquired in falling along arcs CB and DB is equal to the speed that the body would acquire in falling perpendicularly from F to B and from E to B (§444). Therefore, the speeds acquired in falling along these arcs are also to one another as chords CB and DB that subtend them.

456. {Fig. 58} It follows from this that if in circle GB we take arcs B1, B2, and B3, of which the subtending chords are respectively 1, 2, 3, etc., the speeds of a Pendulum that descends successively along arcs 1B, 2B, 3B etc, would be 1, 2, and 3 respectively at point B, that is to say as the chords that subtend the arcs. We can, in this way, give bodies precise and different degrees of speed, and this method is very useful for knowing the laws of collision of bodies, of which I will speak in what follows.

457. {Galileo is the inventor of the Pendulum.} {And Mr. Huyghens of the Pendulum Clock.} Galileo was the first to envision suspending a heavy body by a thread, and by its vibrations measuring time in Astronomical observations and in Physics experiments. Thus, we can regard him as the inventor of the Pendulum, but it was Mr. Huyghens who first used them in the construction of Clocks. Before this Philosopher, measurements of time were very inaccurate or very difficult; but the Clocks that he constructed with the Pendulum give a measure of time infinitely more exact than that which we can gain from the course of the Sun; for the Sun marks only relative or apparent time, and not true time.[3] That is why Pendulum Clocks are sometimes 15 or 16 minutes behind or ahead of the course of the Sun, as I will explain further in detail in talking about Astronomy.[4]

458. Although the vibrations of the same Pendulum in small unequal arcs are made in perceptibly equal amounts of time (§454), nevertheless these times are not equal geometrically. But the oscillations in larger arcs are always made in a slightly longer amount of time, and these small differences that are insignificant in a very short time and in very small arcs, become perceptible once they accumulate over a longer time, or once the arcs differ perceptibly. Now a thousand accidents, be it from cold, from heat, from some piece of dirt that may slip between the wheels of the Clock, can make it so that the arcs described by the same Pendulum are not always equal, and consequently the time marked by the hand of the Clock, measured by the vibrations of the Pendulum, would be either shorter or longer, according to whether the arcs that the Pendulum describes are increased or diminished.

459. Experiment turns out to conform to this reasoning, for Mr. Derham, having made a Pendulum oscillate with circular vibrations in Boyle's machine, found that once the air was pumped from the machine, the arcs that his Pendulum described were a fifth of an inch larger in each direction than in air, and that its oscillations were two seconds per hour slower.

{*Trans. Phil. no. 294*} Once the Pendulum was adjusted in such a way that the arcs that it

[3] See Newton's *Principia*, scholium to the definitions, for the terminology of relative, apparent and true time. See also Brading, 2017, "Time for Empiricist Metaphysics", in *Metaphysics and the Philosophy of Science*, ed. M. Slater and Z. Yudell, Oxford University Press.

[4] Although Du Châtelet never completed later volumes of the *Foundations*, in which such a discussion might have been located, she did complete a translation and commentary on Newton's *Principia*. See Bour and Zinsser, 2009, chapter V.

described were increased by this same quantity of a fifth of an inch in each direction, the vibrations of the Pendulum were slower by six seconds per hour in air; for air slows the movement of Pendulums that much more the larger the arcs they describe.

460. The Pendulum traverses larger arcs in the vacuum for the same reason that bodies therein fall more quickly, that is to say, because there is no air resistance in the vacuum.

461. Mr. Derham noticed furthermore that the arcs described by his Pendulum were a little larger when he had freshly cleaned the mechanism[5] that made it move.

462. {Why Mr. Huyghens thought of making Pendulums oscillate along arcs of a cycloid.} Mr. Huyghens had foreseen these inconveniences and, in order to remedy them and render the clocks as accurate as possible, thought of making the Pendulum that regulates them oscillate along arcs of a cycloid instead of describing arcs of a circle. **{It is because along this curve all arcs are traversed in perfectly equal times.}** For in the cycloid, since all the arcs are traversed in perfectly equal times, the accidents that can change the size of the arcs described by the Pendulum cannot bring about any change in the time measured by its vibrations, once they are made along the arcs of a cycloid.

463. {How the cycloid is produced.} This curve, that is very famous among Geometers owing to the number and the singularity of its properties, is formed by the revolution of any point of a circle of which the entire circumference successively touches upon a right line.[6]

{Fig. 59} When the circle BO successively touches all the points of its circumference along the right line BAb, such that its point B (at which it touched this line at the beginning of its revolution) is found touching the other end b of this line (when the revolution of the circle along this line is complete), we easily see that this line BAb will be equal to the circumference of the circle BO which it was successively touching as if to measure it.

If we conceive now that point B of circle BO, which we call the trace point, leaves a mark of itself at all the points by which it passes from B to b, it will form the curve BGb, and it is this curve that we call *a Cycloid*. The wheels of a carriage trace cycloids in the air as they turn.

464. {Definition.} The circle BO, whose revolution formed the cycloid BGb, is called the generating circle of this cycloid: point G is the summit of the cycloid, and the horizontal line BAb is its base.

[5] The French is "mouvement", referring to the internal mechanism of a clock excluding the face and hands, which is called the movement.
[6] That is, the trace made by a single point of the circle as that circle rolls along a straight line.

465. If we take the generating circle BO as having arrived in its revolution at the point at which its diameter GA divides the cycloid and its base into two equal parts, then this diameter becomes the axis of the cycloid.

466. {Of the properties of the cycloid.} If I wanted to demonstrate to you all the properties of this curve, it would need an entire treatise. I will therefore content myself with indicating to you here those properties that are necessary to the subject that I am treating; you will presuppose demonstrations of them, or if you want to understand these you will find them in the excellent book of Mr. Huyghens, *De Horologio Oscillatorio*, or in the treatise that Mr. Wallis gave on the Cycloid.

1. .**{First property of the cycloid.} {Huyghens, *De Horol. Oscil.*, part. 3, prop. 5, 6 & 7} {Fig. 60}** This curve describes itself by its evolution so that: if CA, CN are two inverted half-cycloids formed by the same generating circle DA, they meet at point C, with their apexes[7] at A and N; and an imagined thread CBA is equal in length to the half-cycloid CA along which I suppose it to have been laid. If we attach to the end of this thread a weight P, this thread will become a Pendulum equal to the half-cycloid CA. Now if this weight P is left to itself, it will fall toward the earth as much as possible by its gravity, and in falling it will straighten out the thread CA, which in straightening out from A to F will, with the end to which weight P is attached, describe a curve AF.

If weight P (which straightened out thread CBA and which brought it to the perpendicular direction CF) continues to move by the action of its gravity, then once it has arrived at F it will describe in reascending from F to N a curve FN, equal to AF; and when point P has arrived at point N, the thread CBP will lie along the half-cycloid CN, to which it is equal. Therefore the entire curve AFN will be described by the evolution and the revolution of the half-cycloid CA, or of the thread CBP which is equal to it. And this curve AFN turns out to be a cycloid equal to the two half-cycloids CA, CN, and to have the same generating circle, and it is consequently twice the length of the thread CBP that is equal to each of the half-cycloids.

In order for the Pendulums to describe cycloidal arcs in their evolution and their revolution, they must be suspended between metal half-cycloids, against which they press continuously as they move, and which prevent them from describing circular arcs.

2. **{Second property.}** The time of fall of a body along any arc of a reversed cycloid is to the time of the perpendicular fall along the arc of the cycloid as half of the circumference of the circle is to its diameter.

{*Idem*, p. 2, prop. 25} It is this property of the cycloid, the demonstration of which you can see in the Treatise of Mr. Huyghens, that led this Philosopher to discover the ratio between the time of an oscillation and the space fallen, of which I have spoken.[8]

[7] These are inverted cycloids so the "apex" is at the bottom.
[8] In the errata, Du Châtelet adds a reference to §328.

3. **{Third property.}** This property of the cycloid gives rise to another: that all the arcs of a reversed cycloid are traversed in equal time by a body that falls along this curve by its own weight; for since by the preceding property the times of the fall of a body along any cycloidal arcs are in a constant ratio to the time of its fall perpendicular along the axis of this cycloid, these times are equal among themselves.

4. **{Fourth property.} {Huyghens, *De Horol. Oscil*., p. 3, prop. 1}** This isochronism of the arcs of the cycloid is based upon a property of this curve that I have not yet spoken to you about, and that is proved by a rather complicated demonstration: every tangent to the cycloid is parallel to that chord of its generating circle that joins the apex of the cycloid to the point of intersection of the line from the tangent point parallel to the base with the generating circle; thus, the tangent HBN is parallel to the chord EA in the cycloid MGL. **{Fig. 61}**[9]

It is easy to see how the isochronism of the arcs of the cycloid follows from this property, even though this is not how it was discovered. For gravity will act on the body at the point of this curve where it finds itself, in the same way that it would act there on the chord of the generating circle that corresponds to this point, since each point of the cycloid has the same inclination as the chord of the generating circle that corresponds to it. Now we have seen that on all the chords of a circle taken from the ends of its diameter, the body receives impulses of heaviness proportional to the chords that it traverses; that is to say, just as much larger as the chords are longer. Thus, in the cycloid each point of this curve having the same inclination as the chord of the generating circle that corresponds to it, the body receives at each of these points impulses of heaviness proportional to the chord, or to double this chord, that is to say, to the arc that remains to it to traverse; for each of these arcs is double the chord of the generating circle that corresponds to it. These impulses are consequently as much the less as the arcs are shorter, and as much greater as they are longer, these arcs being that much more inclined the shorter they are. Following this, two bodies leaving at the same time from points H and B of cycloid FBO[10] with initial speeds proportional to the arcs HF, BF that they have to traverse, would arrive in the same time at point F if they continued to move at the initial speeds from H to F and from B to F with uniform motion.[11] Now, as we can make the same reasoning for all points between H and F, and between B and F, bodies that leave from these different points must reach the apex[12] F in the same time.

I have dwelt upon proving this fourth property of the cycloid, and especially upon showing the Physical reason, because this is what is most useful for the precision of Pendulums oscillating

[9] Fig. 61 in the 1740 edition is misdrawn, as Du Châtelet notes in her errata. See the 1742 edition for the corrected figure. Note that in the redrawn figure the "apex" is the lowest point of the inverted cycloid, as it is for Fig. 62 in both editions.
[10] Corrected from the original (DFO) using the 1742 edition. In Du Châtelet's errata to the first edition, the suggested correction remains wrong (she suggests FHO).
[11] Reference to Fig. 62 added in the errata.
[12] See footnote above for corrections concerning Fig. 61.

in cycloidal arcs.

467. {Fifth property.} I cannot pass over in silence one of the most beautiful properties of the cycloid, and assuredly the one that is the most surprising of all, which is that this curve is the line of fastest descent from one point to another.

468. {The cycloid is the line of fastest descent.} The problem of the line of fastest descent of a body falling obliquely to the horizon by the action of heaviness from one given point to another given point, is famous because of the error of the great Galileo, who thought that this line was a circular arc, and because of the different solutions that the greatest Geometers of Europe have provided for it. One day you will read these solutions in the *Acta Eruditorum* and in the *Philosophical Transactions*, and you will see that all these great men arrived at the same end by different paths, and that they all found that this line was a reversed half-cycloid that had as its origin and its apex the two given points.

469. {This property of the cycloid seems at first to be a paradox.} The solution to this problem seems to be a kind of paradox, since it follows that the straight line that is always the shortest between two given points is not the one that is traversed in the least time. This at first surprises the imagination a little. Nevertheless, geometry demonstrates it, and there is no appeal against it; it depends upon this property of the cycloid, by which the initial speeds of a body at any point on this curve are proportional to the arcs that remain for it to traverse.

470. Thus the line of fastest descent is also that for which all the arcs are traversed in equal times, and it is useful to note that these two properties that manifestly depend upon the same principle (I mean the initial speeds being proportional to the arcs to be traversed) are found to be united in the same curve only when we adopt the system -- or rather the discoveries -- of Galileo on the progression of the fall of bodies.

471. {Solution to the problem of the cycloid by dioptrics given by Jean Bernouilli. *Acta Erudis*, 1697, p. 206.} Mr. Jean Bernouilli, that famous Mathematician[13] who had posed the problem of the line of quickest descent, resolved it by dioptrics, in demonstrating that every ray split in the atmosphere must describe a cycloid; this great Geometer supposed in his solution that light, in traversing mediums of heterogeneous density, must be transmitted along the path of shortest time, as Fermat had asserted against Descartes and as Messrs Huyghens and Leibniz had upheld since Fermat.

472. One can easily feel with what pleasure Mr. Leibniz adopted an opinion that had its source in

[13] In the errata, Du Châtelet changes this to "The famous Mathematician Jean Bernoulli…"

the principle of sufficient reason; for Fermat asserted that since the ray does not go from one point to another either by a direct path or by the shortest one, it befitted the Wisdom of the author of Nature that it go by the path that it traverses in the least time possible.

This is not the place to enter into this discussion. You can see what Mr. de Mairan reported about the dispute between Descartes and Fermat in the *Mémoires de l'Académie des Sciences Année 1722* while you wait for me to talk to you about it, when I will explain to you the refraction of light.

473. You have seen above that in order for a Pendulum to describe cycloidal arcs, it is necessary that it be suspended between two half-cycloids (as in Fig. 60) which, being usually metal, prevent it from describing a circular arc.

{Fig. 63.} Now, although the two half-cycloids CA, CN prevent body P from describing circular arc EFL, nevertheless there is near the apex of the cycloid a small space PFP in which the Pendulum moves in the same way as if it was oscillating freely in circle EFL. This is the true reason why oscillations of the Pendulum in different very small circular arcs are nevertheless completed in perceptibly equal times, as I have said.

And this is why we seldom suspend large Pendulums between cycloidal arcs, the smallness of the arcs that they describe sufficing to render their vibrations isochronous, and it is only for small Clocks whose Pendulum is very short that we make use of the cycloid.

474. {Ratio of the time of one oscillation to the time of the vertical fall along the half-length of the Pendulum.} {Fig. 63.} It follows from the equality of the small circular arc PFP with this portion of the cycloid AFN that: the time during which a body makes one oscillation in a very small circular arc is to the time of the perpendicular fall along the half-length of the Pendulum as the circumference of the circle is to its diameter, since the time of one oscillation in a cycloid follows this proportion.

This equality between the time of the oscillations in a small circular arc and the time of the oscillations in small cycloidal arcs was necessary for finding, for deducing from it, as Mr. Huygens did, the distance terrestrial bodies traverse due to gravity, in the first second of fall, when falling towards the Earth; for Pendulums that make their oscillations solely by the force of gravity describe circular and not cycloidal arcs.

475. The time of the oscillations of two Pendulums that oscillate in similar circular arcs is as the square root of the lengths of these Pendulums.

You have seen in chapter 13 (§315, no. 4) that a body that falls toward the earth by the force of gravity alone traverses spaces that are as the squares of the times taken to fall, or of the speeds acquired in falling, at the end of each of these times.

Now in the oscillations of Pendulums the spaces traversed are circular arcs, whose radii are the lengths of the Pendulums. Thus, the time of fall along arc EB is to the time of fall along the similar arc GD as the square root of EB to GD, and consequently as the square root of AB to CD, for the arcs are to one another as their radii. **{Malézieu, Book 8, Corollary of the Prop. 5.}** **{Fig. 64.}** We easily see that what is true for the half-oscillations EB, GD, is also true for the whole oscillations EBF, GDH. **{The lengths of Pendulums are to one another as the squares of the times of their oscillations in similar arcs.}** Thus, the lengths of the Pendulums that describe similar circular arcs are inversely proportional to the square of the ratio of the number of their oscillations, in equal times. Consequently, Pendulum AB, which is 9 feet long, say, will make two oscillations in the same time that Pendulum CD, 4 feet in length, makes three. For the squares of these oscillations are 9 and 4 respectively, which is the length of the Pendulums. The vibrations that are made in cycloidal arcs follow the same proportions.

476. It follows from this that in Pendulums that oscillate in similar circular arcs, the longest are those whose oscillations are the slowest; for they move along a similar arc, and at a greater incline than shorter Pendulums. Therefore, a Pendulum that makes its vibrations in one second must have a certain determinate length, since the length of Pendulums decides the time they take to make their oscillations.[14]

477. {Length of the Pendulum that beats the seconds in Paris, determined by Mr. Picard.} Mr. Picard had determined the length for the Pendulum that beats the seconds in Paris to be 3 Paris feet and 8 half lines. It was this length and proportion that Mr. Huyghens had found between the time of an oscillation and the quantity of vertical fall (§328) that gave Mr. Huyghens the idea of making the length of the Pendulum that makes its vibrations in one second in Paris a universal measure for all countries and for all times; and to render this measure univocal, he gave the name *horary foot* to one third of this length. **{Universal measure proposed by Mr Huygens; this is called *horary foot.*}**

478. But in order for this measure to be universal, heaviness would have to be the same at all points on the surface of the earth. For heaviness being the sole cause of the oscillation of Pendulums (§444) and this cause being assumed to remain the same, it is certain that the length of the Pendulum that beats the seconds must be invariable, since the duration of the vibrations depends upon this length and upon the force with which bodies fall toward the earth; and that consequently the measure that results from this would be universal for all countries and for all times, for we do not have a single observation that can lead us to believe that the action of gravity is different in the same places at different times.

[14] A better, though less literal, translation reads: "Therefore, it must be the case that a Pendulum with a period of one second has a certain determinate length, since the length of a Pendulum decides its period."

479. {This measure cannot be universal, and why.} One must admit that this idea is very nice, and that a universal measure would be very desirable, but the assumption necessary to make it so -- that is, equal heaviness in all regions of the earth -- is found to be entirely false. For, some incontestable observations have shown that the action of heaviness is different in different regions,[15] and that toward the Pole one must always lengthen the Pendulum, and toward the Equator shorten it, in order for it to make its vibrations in equal time. Thus, this measure proposed by Mr. Huyghens cannot be universal for[16] all places on earth, but only for the countries situated on the same latitude as Paris, since it is in Paris that the length of the Pendulum that beats the seconds was determined. To render this measure universal one would need to obtain, via experiment, tables of the differences in the lengths of the Pendulum that would beat the seconds at the different latitudes in the two hemispheres, just as we have obtained them via theory for our hemisphere; and by comparing all these lengths with the length of the Pendulum that beats the seconds in Paris, this would serve also to determine the shape of the earth (§377).[17]

This is a project whose execution would be useful for Physics in more than one way, but these operations need very practiced hands and very attentive minds, and it is not at all easy to determine these lengths by experiment with the precision necessary to discern differences that depend sometimes on less than a quarter of a line.[18]

480. To achieve this, above all the length of the Pendulum that beats the seconds at a certain latitude must have been securely established, and this is what we can pride ourselves on having for the latitude of Paris since the experiments to determine it that Mr. de Mairan carried out in 1735.

Mr. Picard and Mr. Richer had already given this length, but in matters that depend upon experiment, it is not sufficient to be right, one must be very sure of being right, and before 1735 we did not yet have for the length of the Pendulum this sort of certainty that leaves nothing to be desired.

481a.[19] {How to find out the length of the Pendulum that beats the seconds in any given place through the force of heaviness alone.} To know the quantity of the action of heaviness in a certain place, it is not sufficient to have a Pendulum Clock that beats the seconds with accuracy

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[15] The French original "dans différens climats" might better be translated as "in different climes", where climes are regions differentiated by climate.
[16] This is corrected from "par" to "pour" in the errata.
[17] The errata changes "by comparing" to "to compare", so that the translation would better read: "... our hemisphere, and to compare all these lengths with the length of the Pendulum that beats the seconds in Paris, which would serve also to determine the shape of the earth (§377)."
[18] i.e. approximately a fiftieth of an inch.
[19] There are two paragraphs marked "481". This is not corrected in the second edition. We here label them 481a and 481b.

in that place; for it is not heaviness alone that moves the Pendulum of a Clock but the action of the spring, and in general the entire mechanism acting on it and combining with the action of gravity to move it. It is a very difficult and very delicate problem to determine, for a Pendulum Clock, how much the length of the pendulum that counts the seconds in this Clock is altered due to the construction of the Clock, as compared to that of a pendulum that makes its oscillations in the same time due to the action of heaviness alone. Nevertheless it is this length that must be found in order to know the quantity of the action of heaviness alone in the place for which one wants to determine the length of the seconds Pendulum.[20]

To achieve this we use a heavy body suspended by a thread, which being pulled from its point of rest, oscillates in small circular arcs by the action of heaviness alone. To know how much this pendulum oscillates in a given time, we use a Pendulum Clock that is well regulated to mean time[21] and that beats the seconds of this time very exactly. We then count the number of oscillations made by the Pendulum upon which heaviness alone acts (which is called the *Experimental Pendulum*) while the Pendulum Clock beats a certain number of seconds. And since the number of oscillations that the pendulums make in equal time is as the inverse square root of their lengths (§475), once we know the number of oscillations that two pendulums make in a given time, we know the ratio of their lengths by squaring these numbers. Thus the squares of the oscillations that the Pendulum Clock and the experimental pendulum make in equal time give the relationship between the length of the experimental pendulum and that of the simple pendulum that would make its oscillations through the force of heaviness alone, would be isochronous with the compound pendulum of the Clock, and would consequently beat the seconds in the latitude where we do the experiment. And this is the pendulum length we are looking for.

481b. {Determination of the length of the Pendulum that beats the seconds in Paris, by Mr. de Mairan in 1735.} It is in this way that Mr. de Mairan determined the length necessary for the pendulum to beat the seconds in Paris by the action of heaviness alone, as 3 feet $8\frac{17}{30}$ lines, or about $\frac{5}{9}$ of a thread of agave (thread taken from the leaf of a type of aloe) almost as fine as a hair, and from which was suspended a copper ball one inch in diameter.

482. This length is about midway between those which Messrs Picard and Richer had given; and if we take it as 3 feet $8\frac{5}{9}$ lines, it is the same as that which Mr. Newton reports in the third Book of his Principles, following the measurements of Messrs Varin and des Hayes taken in 1682.

483. One can see in the excellent Memoir of Mr. de Mairan all the precautions he took to be sure

[20] i.e. a pendulum that beats seconds under the force of gravity alone.
[21] The French reads: "bien reglé sur le tems moyen".

of the accuracy[22] of his experiments, and one will see that the wishes of those who only take the trouble to wish cannot do more.

{It is to this length that the Academicians who went to the Pole, and to the Equator, referred their observations on the Pendulum.} It is to these measurements that the Academicians who had measured a degree of the Meridian below the equator, and in the arctic circle, referred all the observations they made on the length of the Pendulum, in these different regions.[23]

484. All that I have said up to now about Pendulums must be understood to apply only to Simple Pendulums, that is to say, Pendulums having a single suspended weight, and whose thread is assumed to be without heaviness. For when the thread to which the weight is attached has a heaviness perceptible in relation to this weight, then the Simple Pendulum becomes a compound pendulum (§449), since the weight of the thread that must then be taken into account has the same effect as a second weight pulling on the same thread, and Compound Pendulums are nothing other than Pendulums to which several weights are attached at invariable distances from each other and from the point of suspension.

485. {Of Compound Pendulums.} Compound Pendulums follow the same laws as Simple Pendulums, but they follow them with certain modifications.

486. To determine the time of the oscillations of a Compound Pendulum and the arcs that it describes, we must consider one thing that I have not yet spoken about, because it relates principally to Compound Pendulums; that is, the *center of oscillation*.

487. {Of the center of oscillation.} The center of oscillation of a Compound Pendulum is the point at which the efforts or actions of the weights that compose it come together to make this Pendulum make its vibrations in a certain time; thus, the center of oscillation and the center of gravity have a necessary relationship.

488. **{Of the center of gravity.}** We call *center of gravity* the point through which the line that would divide the body into two equally heavy parts necessarily passes, such that if each half was placed in the pan of a balance, they would remain in equilibrium.

489. All the gravity of a Body can be conceived of as gathered together at a single point, such that the other parts are considered as being entirely deprived of it, and it is thus that we conceive of the heaviness of simple Pendulums.

[22] The French term here is *justesse*.
[23] See also §479.

490. The center of gravity of a Body always lies along a line perpendicular to the horizon, such that this Body can be maintained there, whether it is suspended from the point itself of its center of gravity, or from any point along this line that we call the *line of the center*.

491. The center of oscillation is always on this line of the center of gravity.

When two or more bodies remain together, whether they are contiguous or separated, they have a common center of gravity; this center is a point on the straight line joining the centers of these bodies, and this point is always situated in such a way that the distance of the bodies to this point is always in inverse ratio to their gravity.

492. {Of the center of oscillation of Simple Pendulums whose thread is without perceptible weight.) The center of oscillation of a simple Pendulum whose thread is assumed to be without heaviness (which is ordinarily the case) is not at all at the point of its center of gravity, as one would at first think. Rather, it is on the line of the center of gravity a little lower than the point of the center. It is further away or closer according to a certain proportion between the radius of the pendulum ball and the length of the thread to which it is attached. This is because the distance from the center of gravity of the ball to the point of suspension must be taken into account; for, the length of the thread remaining the same, this distance will be greater as the radius of the ball is greater, and vice versa. It is to Mr. Huyghens again that we owe this observation, and it is he who determined this proportion between the radius of the ball and the length of the pendulum to find the center of oscillation.

493. The true length of the Simple Pendulum, whose thread is assumed to be without heaviness, is therefore not the length of the thread from the point of suspension to the point at which the ball is attached, nor to the center of gravity of this ball. Rather, this length is to be counted from the point of suspension to the center of oscillation, which is the same as the center of gravity only when the length of the thread at a certain point exceeds the radius of the ball; for then the lowering of the center of oscillation becomes imperceptible and is no longer to be counted.

494. {What is the center of oscillation of a Simple Pendulum when the thread has a perceptible weight.} When the thread of the Simple Pendulum has a heaviness that is perceptible in relation to that of the weight attached to it, then this Pendulum is no longer considered to be a simple pendulum, but a composite pendulum (§484). Its center of oscillation is then no longer in the suspended ball; it is on the thread itself at some point above this ball, that is to say, at a point where we conceive the action of the gravity of the thread, and of the weight, coming together. This point is higher as the weight of the thread is greater in relation to that of the ball, and vice versa.

In this case, the true length is the distance between the point of suspension and the center of oscillation. The oscillations of this pendulum will be quicker than if the thread were without heaviness; for then the true length of the pendulum would be less (§476).

495.[24] **{How one finds out the center of oscillation of a Compound Pendulum.}** We saw (§476) that a weight suspended by a thread makes oscillations that are as much slower as the thread is longer, or equivalently, as the body is further from the point of suspension, and vice versa. Thus {Fig. 65} if, for example, to a four foot long thread CA that carries a weight P at its end A, we add a second weight B one foot higher at point Q (that is to say, at 3 feet from the point of suspension), then body P (that is four feet from the point of suspension) must make slower oscillations than body B (that is only three feet away). However, since these two weights are hanging by the same thread, this thread cannot make longer and shorter vibrations at the same time; it will therefore make them in a time that will be the midpoint between the slowness with which it would have oscillated if weight P attached at four feet from the point of suspension had been there alone, and the speed that the oscillations would have had if there had only been weight B attached at Q. Thus, the second weight speeds up the vibrations of the first, and the first slows down those of the second, and the center of oscillation of this pendulum will be at the point at which, if these weights were brought together, the Simple Pendulum that they would then compose would make its vibrations in a time equal to the time of the vibrations of the Compound Pendulum to which they are attached separately. Thus, to seek the center of oscillation of a Compound Pendulum is to seek the length of a Simple Pendulum that would make its vibrations in a time equal to those of this Pendulum, and the true length of the Compound Pendulum is that of the Simple Pendulum that would be isochronous with it, as pendulum CR, for example, is to Pendulum COA. Now, since the lengths of the Pendulums are as the squares of the times of their oscillations, one easily sees {Fig. 65} that the Simple Pendulum CR, whose vibrations would be isochronous with those of the Compound Pendulum COA, would be more than three feet long, and less than four, since its oscillations would be neither as slow as those of the weight attached at four feet, nor as quick as those of the weight attached at three feet. Consequently, a Simple Pendulum is always shorter than the Compound Pendulum with which it is isochronous, and the center of oscillation of the Compound Pendulum COA will be between the two weights P and B, that is to say, at about point O.

496. One sees from this that to determine what happens to compound pendulums, we must decompose them; for we can see objects only in parts, and to consider the composite we must always simplify it.

497. One easily sees that in pendulum COA, composed of two weights, the closer one of these

[24] In this paragraph we have incorporated corrections from the errata.

weights is to the point of suspension, that is to say the further the two weights are from one another, the closer the center of oscillation is to the point of suspension, and vice versa, such that if these two weights were equally distant from the point of suspension, their centers of oscillation would combine, and the compound pendulum would become a simple pendulum, since the simple pendulum that would be isochronous with it would be of the same length.

498. Thus, any pendulum from which a single weight is suspended can be considered as a compound pendulum, by supposing that the suspended weight is divided into several parts whose different gravities are combined at the center of oscillation of this pendulum.

499. All that has been said about a compound pendulum with two weights can be said about a pendulum composed of three, or four, or any number of weights; for the proportions are always inviolably the same.

500. {The weight and the matter of the bodies that make up the pendulum are irrelevant. And that is because gravity is proportional to mass.} In all that I have said to you about pendulums in this chapter, I have specified neither the weight nor the type of the bodies suspended. For -- air resistance being almost imperceptible on the pendulums, and gravity being proportional to mass -- all bodies, of whatever type they may be, make their vibrations equally fast, all else being equal. This is further proof that gravitation acts according to the direct quantity of the proper matter of bodies (§361),[25] for all truths mutually support one another.[26]

[25] The reference should be to §362.
[26] The French reads: "se donnent mutuellement la main."