Vanishing Matter and the Laws of Motion

Descartes and Beyond

Edited by Dana Jalobeanu and Peter R. Anstey

Routledge
Taylor & Francis Group
New York London

7 On Composite Systems

Descartes, Newton, and the Law-Constitutive Approach

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GENERAL INTRODUCTION

The title of this volume is *Vanishing Matter and the Laws of Motion*. The context is the early modern debate over how best to revise or replace the Aristotelian account of individual bodies as the things of which the world is constituted. In the context of Newtonian mechanics, the phrase "vanishing matter" refers to the view that this theory provided a dynamical account of the behavior of large-scale material bodies, while at the same time treating them as mathematical entities and providing no insight into their nature. There is something right about this: Newtonian mechanics enables us to treat the behavior of bodies without first saying anything about their metaphysical nature. This signals an important shift in the relationship between dynamics and matter theory. However, the phrase "vanishing matter" implies the vanishing of matter theory from physical theory, as though Newtonian mechanics is silent about metaphysical questions concerning the nature of material bodies. I think there is a different way to understand the shift that took place. Far from being silent, Newtonian mechanics has significant implications for matter theory. With hindsight, we can see that it is not that the traditional questions of matter theory vanish, but rather that the development of Newtonian mechanics enriches the logical and philosophical space in which matter theory is to be explored, profoundly changing the framework within which these issues are to be addressed. In particular, various metaphysical aspects of matter theory—such as whether bodies have actual parts—become entangled with (rather than being independent of, and prior to) the details of the physics. This marks a deep change in the relationship between physics and metaphysics, and one to which any later attempt to do matter theory must pay due attention: the philosophical space changes with the advent of Newtonian theory, and there is no going back.

That is a big claim, and very general, so now to specifics. I have one very narrow line of argument that I want to push, and it concerns one way to read the implications of the treatment of bodies in Descartes and in Newton. I will argue that the extension of this treatment to composite systems reveals important consequences for matter theory. I will begin with a brief discussion of bodies in Descartes's system, by way of introduction, and this will enable me to set up the issues concerning composite systems that I want to focus on in this chapter.

SPECIFIC INTRODUCTION

In his *Principles of Philosophy* (1644), Descartes offered his three laws of nature, concerning the behavior of "things" and of "bodies." Here are the laws as he stated them in his *Principles* (Part II, arts. 37, 39, and 40):

> The first law of nature: that each thing, as far as is in its power, always remains in the same state; and that consequently, when it is once moved, it always continues to move.
>
> The second law of nature: that all movement is, of itself, along straight lines; and consequently, bodies which are moving in a circle always tend to move away from the center of the circle which they are describing.
>
> The third law: that a body, on coming in contact with a stronger one, loses none of its motion; but that, upon coming in contact with a weaker one, it loses as much as it transfers to that weaker body.
>
> (Descartes 1991: 59, 60, 61)

But what are the "things" and "bodies" to which these laws apply? If Descartes's laws are to say anything, then there must be bodies to which they refer. Call this the "problem of bodies." For Descartes, the answer is "parts of matter." Famously, however, this answer masks a difficulty that Descartes never satisfactorily resolved. In this section I will briefly review what this difficulty is and how it arises, and outline one possible response, which I call the "law-constitutive" approach.[1] With this in place, I will then turn attention to the main purpose of this chapter: the application of the law-constitutive approach to composite systems.

According to Descartes, on the one hand we have a clear and distinct idea of matter as extended, and on the other hand experience teaches us that this extension is divided into parts, having various shapes and motions. If our metaphysics is to be founded on clear and distinct ideas *and* to include parts of matter, then we had better have a clear and distinct idea of those parts. For *this* to be possible, Descartes must provide *within his metaphysical system* the resources for dividing matter into parts *such that* we can clearly and distinctly perceive that it is so divided.[2] The answer that Descartes appears to give is that *motion* is the principle by which matter is divided into parts. In *Principles* II, art. 25 Descartes gives his definition of "What movement properly speaking is," and then offers an account of the division of indefinite extension into parts or bodies *through* motion:

one body, or one part of matter, is everything that is simultaneously transported. However, motion is itself defined by appeal to the parts of matter. The resulting view is that motion is defined in terms of bodies, but the division of indefinite extension into bodies is achieved through their relative motions. This is, at best, a rather tight circle. Whatever you might think about this, Descartes's next move is to present his laws of motion and, as we have seen, these refer to bodies. The difficulty we are faced with is that we have laws that refer to bodies while not yet having in hand a completed account of bodies.

There are two ways to respond to this difficulty.[3] On the one hand, you might attempt to "complete" the metaphysical account of bodies, providing criteria of individuation and identity that enable a solution to the problem of bodies prior to the specification of the laws of nature. On the other hand, you might suggest that the laws themselves contribute to the solution of the problem of bodies, such that bodies *are*, in part, whatever satisfy the laws. We expand the rather tight circle where motion and body are inter-defined, and thereby hope to turn a vicious circle into a virtuous one. This is what I call a "law-constitutive" approach to the problem of bodies.

I have argued in detail for this approach to the problem of bodies elsewhere (Brading forthcoming), where I also show that the law-constitutive approach was explicitly adopted by Newton, for whom a necessary condition for something to *be* a physical body is that it satisfy the laws. This claim runs at least from "De Gravitatione" (where his account of bodies as impressed shapes in space includes the requirement that these shapes move according to the laws), to drafts made in preparation for the third edition of the *Principia*. I will not argue for this here. Rather, my goal is to extend the law-constitutive approach to the explicit consideration of composite systems. Once again, I think it is helpful to start from Descartes, and then move to Newton. I will say something later about the extent to which I am willing to argue for the law-constitutive approach to composite systems as an interpretation of either Descartes's or Newton's own views, but my main purpose is not exegesis. Rather, my interest is in how the philosophical landscape of matter theory is changed by the philosophical moves that Descartes and Newton make, and my point will be to display some of the rich and far-reaching metaphysical implications of the approach.

The topic of composite systems has two aspects: (1) composite systems constructed from bodies, and (2) the question of whether those bodies themselves should be regarded as composite systems. In each case, there are metaphysical and physical questions that one can ask. With respect to (1), we should distinguish between such metaphysical questions as "*In virtue of what* is the result a *composite system* rather than merely a *collection* of bodies?" or "What is the principle of unity here?" and physical questions such as "What is the glue that binds the bodies together into a composite system?" As regards (2), when we ask about the dividing of bodies and of composite systems into parts (and thus about the status of the bodies themselves), we should distinguish between metaphysical divisibility and mere physical divisibility. In both cases, (1) and (2), my concern is with the metaphysical questions, and not the physical.

In what follows, I begin with the construction aspect (1), first in Descartes and then in Newton. I argue that basically the same principle of unity emerges from both Descartes's and Newton's work, addressing the metaphysical question "*In virtue of what* is the result a *composite system* rather than merely a *collection* of bodies?" The remainder of the chapter discusses the division aspect (2), where my focus is on the actual and potential parts debate. I suggest that neither Descartes nor Newton is best understood as ascribing to either doctrine, but rather that their work marks an important shift in the philosophical framework within which the issue of divisibility should be addressed.

FROM BODIES TO COMPOSITE SYSTEMS IN DESCARTES

I will begin with the construction project (1) as it appears in Descartes and in Newton,[4] and I will argue that the same kind of answer to the metaphysical "in virtue of what" question emerges from both Descartes's and Newton's work.

When considering Descartes's approach to this issue, it is worth starting from the laws of nature that he presents in his manuscript *The World* (c. 1633; Descartes 1998). There are important differences between the cosmological projects set out in *The World* and the *Principles*,[5] and between the two versions of the laws, but I think that the "in virtue of what" question receives essentially the same answer. This answer is immediately evident in *The World*, but is somewhat masked by the changes to the laws that Descartes makes in the *Principles*, and for this reason it is helpful to begin with *The World*.

Descartes begins with a conservation law for the behavior of a lone body, free from collisions with other bodies. He writes:

> The first is that each individual part of matter continues always to be in the same state so long as collision with others does not force it to change that state.
>
> (CSM 1 93)

What we need now is an account of what it is to stay in the same state, of what it is to change state, and also of under what conditions change so defined can take place. Descartes continues as follows:

> That is to say, if the part has some size, it will never become smaller unless others divide it; if it is round or square, it will never change that

> shape unless others force it to; if it is brought to rest in some place, it will never leave that place unless others drive it out; and if it has once begun to move, it will always continue with an equal force until others stop or retard it.
>
> (ibid.)

Adopting the law-constitutive approach, a necessary condition for the individuation and identity of a part of matter, or a physical body, is that when it is free from collisions, it retains the same shape, size, and quantity of motion. I have argued elsewhere for this approach to individual bodies as a solution to the problem of bodies in Descartes (Brading forthcoming), and will not do so here. Rather, I will move directly on to the consideration of composite systems.

Having stated his first law, the next step in Descartes's project is to move from the consideration of an isolated individual body to an analysis of what would happen if a second body was added to the conceptual structure. The second law of *The World* reads:

> I suppose as a second rule that when one body pushes another it cannot give the other any motion unless it loses as much of its own motion at the same time; nor can it take away any of the other's motion unless its own is increased by as much.
>
> (CSM 1 94)

This is a law of conservation of the total quantity of motion of a composite system: it extends the first law from the single body case to the case of a pair of bodies, and provides us with a law for a composite system of colliding bodies considered as isolated from the rest of matter.[6]

At least, that is how I think we should read it. Interpreting the second law as a conservation law follows one of the two main lines of interpretation in contemporary literature.[7] The other standard interpretation treats the second law as a law of impact, judging it by its success at determining the outcome of collisions.[8] But this law is not sufficient to determine the outcome of a collision because it does not determine how the total quantity of motion will be distributed among the component bodies after the collision and it says nothing about the subsequent directions of these component bodies. Viewed in this way, the law is a failure.[9] However, viewed as a conservation law it achieves a very important goal, by generalizing the first law for single bodies to the case of a pair of interacting bodies: a composite system (not subject to collisions from outside) satisfies conservation of quantity of motion just as a lone body (not subject to collisions from outside) does. This is a global claim about the composite system as a whole, and not a claim about its parts.[10]

Our task now is to adopt the law-constitutive approach with respect to Descartes's laws, as presented in *The World*, and to examine the implications (if any) for the metaphysics of composite systems. It seems to me that we can draw three important conclusions:

- First, just as the first law gives a necessary condition for the individuation and identity of bodies, the second law generalizes this condition to composite systems.
- Second, satisfaction of the second law is (partially) constitutive of what it is to be a composite system: when it is free from outside collisions, is conserves its total quantity of motion. It is a composite because there were two bodies initially, and it is a whole because the composite satisfies the conservation law. Thus, the second law provides a *principle of unity* in virtue of which the composite is a genuine whole.
- Third (and this is a negative conclusion), the laws of nature offered by Descartes in *The World* cannot be used to individuate the component bodies of a composite system. The second law is silent on the behavior of the components considered individually, and while the third law ascribes a tendency to the components, this is not sufficient to determine the behavior of the components within the composite system. The upshot is that, on this approach, there *are* no determinate components.

The positive proposal here is that the laws offer a principle of unity in virtue of which a composite forms a genuine whole as opposed to a mere collection. Specifically, the composite conserves its total quantity of motion, and the claim is that this is a necessary and sufficient ground for a genuine unity.[11] However, on the negative side, this genuine unity lacks determinate components.

The failure to determine the redistribution of the quantity of motion among the component bodies following collision is something that Descartes seeks to address in his revision of the law in the *Principles* (where it appears as the third law, see above) and the accompanying rules of collisions. This law, unlike that appearing in *The World*, is directed at the behavior of the parts of a composite system. Together with the rules of collision (*Principles* II, arts. 46–52), it seeks to determine how the motion of the component bodies of is affected by a collision. Note that this determination remains subject to the *global* constraint on the composite system as a whole that the total quantity of motion of the whole remain unchanged.[12] The first law of the *Principles* remains essentially the same as that of *The World*, for our purposes, and the old third law of *The World* now becomes the second law of the *Principles*.

Viewed from the law-constitutive perspective, we can say that the *Principles* attempts a significant step forward: in addition to providing a principle of unity for composite systems free from outside collisions, the third law and the rules of collision can also be used to try to determine the behavior of the component bodies of a composite system, and therefore to provide a necessary condition for the individuation and identity of the components: a necessary condition for something to *be* a component body of a composite system is that it move according to the third law and the rules of collision.

By now we have moved far from Descartes exegesis: throughout his statement of the laws and the rules of collision, Descartes writes as if the bodies that are their subject matter are already given. I have said that Descartes has not, in fact, succeeded in providing the bodies that are the subject matter of his laws, prior to his statement of the laws, and I will come back to this point later on (see ‘From bodies to their parts’, below). What I am doing here is adopting one possible solution to this problem and re-interpreting Descartes’s laws in this light with a view to displaying the philosophical consequences for matter theory. Viewed through the lens of this law-constitutive approach, the following points emerge:

- First, the laws of Descartes’s *Principles* provide (or attempt to provide) necessary conditions for the individuation and identity of not just isolated bodies, but also isolated composite systems, and the component parts of those systems.
- Second, the third law provides a principle of unity for composite systems: a composite system is a unified whole *in virtue of* conserving its total quantity of motion.
- Third, insofar as the third law and the rules of collision fail to solve the problem of collisions, they are also insufficient to determine the component bodies of a composite system, and we are still left with a composite system consisting of indeterminate parts.
- Fourth, as regards the conditions placed on them by the laws, the *ontological status* of isolated bodies, isolated composite systems, and component parts of isolated composite systems are much on a par with one another. This is a point we shall return to later when we consider the status of the parts of bodies. Descartes has available a criterion for answering the metaphysical question “In virtue of what is a given entity a genuine unity?” for isolated bodies and composite systems, including the universe as a whole, and that answer is “in virtue of possessing a constant total quantity of motion.” An advantage of this approach is that we get a unified approach to individuation through conservation of total quantity of motion, all the way up to the cosmos as a whole.

With this application of the law-constitutive approach to Descartes’s system in mind, I want to turn our attention now to Newton, and to his construction of composite systems, during which I will draw some conclusions for matter theory.

FROM BODIES TO COMPOSITE SYSTEMS IN NEWTON

I am going to start from the assumption that Newton explicitly proposed a version of the law-constitutive view according to which a necessary condition for an entity to be a physical body is that it satisfy the laws of motion. As noted above, I have argued for this elsewhere and my goal here is to extend this approach to composite systems.

Just as for Descartes, Newton’s first law concerns the behavior of a single isolated body. The question then arises: how does Newton progress from the motion of a single isolated body to the behavior of interacting bodies?

Newton's general strategy in the *Principia* is exactly that found in Descartes: we proceed by construction from the behavior of isolated individuals to the behavior of composite systems via conservation laws. But in Newton the strategy is implemented with clear success when it comes to the component parts of composite systems. From the beginning of his consideration of individual bodies, Newton is interested in saying precisely how the state of a body changes as a result of a collision. Newton's second law tells us in what way a body's state will be changed by the action of an external force, and, crucially, this change is quantifiable. It is the third law, however, that allows Newton to extend his analysis to the behavior of bodies interacting with one another. By means of his third law, Newton achieves an answer to the distribution question and an extension of the conservation of the linearity of motion from single bodies to composite systems;[13] his solution provides a rule that determines uniquely and quantifiably the outcome of two body collisions and interactions.[14]

From the law-constitutive perspective, this is important not just because it solves a problem in mechanics, the problem of collisions, but, more fundamentally, because it extends the law-constitutive approach to the *component bodies* of a composite system. Putting the point more dramatically: it gives necessary conditions for something to *be* a part of a composite system, *and sufficient conditions for those parts to be determinate*. Thus, this solves a problem in physics, but also—when viewed from the law-constitutive perspective—a problem in metaphysics.

If we adopt the law-constitutive approach, we can draw the following conclusions:

- First, Newton's laws provide necessary conditions for the individuation and identity of bodies, composite systems, and the component parts of those systems.
- Second, the laws provide a principle of unity for composite systems. The role of the third law is to determine the behavior of component bodies of a system, behavior that must be consistent with the first law continuing to hold for the composite interacting system as a whole. In other words, an analogous principle of unity for composite systems that we drew from Descartes's system is also available in Newton's system: conservation of quantity and direction of motion of the whole, when free from external interactions.
- Third, the laws are sufficient for the parts of a composite system to be determinate.
- Fourth, we note—as we did when considering Descartes's system—that the ontological status of isolated bodies, isolated composite systems, and component parts of composite systems is equal. This approach does not deliver any account of ontological priority of bodies over systems, or vice versa. Rather, they are all on a par.

As I stressed at the outset, my goal is not Newton exegesis but rather the question of how best to think about bodies and composite systems in the light of the legacy left to us by Descartes and Newton. But let me be clear about how far I am willing to support what I have said as exegetical. First, I *do* think that the law-constitutive approach to bodies is explicit in Newton, as I argue in Brading forthcoming. Second, I *do* think the constructional strategy for how to build composite systems out of bodies is explicit in Newton. The argument for this is set out in the Appendix, both with respect to how Newton presents his theoretical system and also with respect to how he applies it. Finally, while I do *not* think that the law-constitutive approach to composite systems is explicit in Newton, I *do* think it follows very naturally from the conjunction of the law-constitutive approach to bodies plus the constructional strategy, both of which I maintain are explicit in Newton.

THE TRANSFORMATION OF MATTER THEORY

My claim is that the law-constitutive approach to the construction of composite systems from bodies leads to important metaphysical results traditionally associated with matter theory. First and foremost, it provides a principle of unity in virtue of which a composite system constitutes a genuine whole rather than a mere collection. This principle of unity is not about merely *physical* unity. It is not, for example, about the glue that binds a composite system together (for this, on the Newtonian picture, we need specific force laws). Moreover, the unity of the bodies from which the composite is made is itself grounded in the very same principle. The conservation of quantity of motion by a body, or by a composite system, should be read as a *metaphysical* principle, the necessary and sufficient ground of the unity of the body or system. This proposal for a principle of unity can be challenged, of course, but it should be challenged as a metaphysical claim about matter theory, and thus duly recognized as such.

In the light of this, what should we say about the apparent absence of matter theory in Newtonian mechanics? I think we have an alternative account of why Newtonian mechanics appears to be silent about matter theory. It is not simply that bodies are being treated mathematically, and that this can be done without *first* providing a theory of matter. This is true, but it is not the whole story. It is *not* that we do not *have* to provide a matter theory first, it is that we *cannot*: on the law-constitutive approach, the matter theory comes along with the laws. The laws give necessary conditions on *what it is* to be a body, and on *what it is* to be a composite system of bodies. Furthermore, as I shall now argue, the laws give necessary conditions on *what it is* to be a *part* of a body. Traditionally, these questions belong to matter theory and to metaphysics, but with the development of Newtonian mechanics I think that the two become entangled. It is not that matter theory vanishes, but that it is no longer prior to mechanics. In the next section, below, we will see how this plays out when it comes to the debate over the status of the parts of bodies.

Clearly intertwined with the story I have told is the search for the laws of collision. In her chapter in this volume, Jalobeanu returns to the developments that took place historically between the proposals of Descartes and Newton, focusing on the largely forgotten contributions of William Neile. At the time, the challenge posed by the problem of collisions was seen as twofold: (1) to search for the properties by which to characterize bodies so that the problem of collisions can be solved, and (2) to "account for" those properties in terms of something else. This "accounting for," on the Cartesian model, was the reduction of the dynamical properties (such as hardness and elasticity, for example) to the geometrical properties of size, shape, and motion (this being what Jalobeanu calls the "strong program" of Cartesian geometrical reductionism). Thus, according to Jalobeanu, Neile repeatedly expresses his concern that the problem of collisions is not adequately solved until we have given a definition of the basic concepts used in our laws, including "hardness" and "elasticity," *prior to and independently of* our specification of the laws: matter theory is prior to physics.

Jalobeanu's chapter beautifully illustrates the tension between the inherited view that matter theory is prior to physics (such as is exemplified by the Cartesian "strong program") and the newly emerging law-constitutive approach. It seems to me that we can see the protagonists in the collisions debate wrestling with this very issue. The question is: if we have a solution to (1), what more could we possibly want? More precisely, what is it that we asking for in (2), and what work can this "something more" be made to do?[15] The lesson we should take away is: nothing. A *complete* characterization of the properties of bodies can, in principle, be given by the laws: there are no "residual" questions that a separate matter theory should address; matter theory is absorbed into physics. This is a profound shift in the relationship between physics and metaphysics, the seeds of which were sown by Descartes, and which forms part of our inheritance from Newtonian natural philosophy. As Murray, Harper, and Wilson argue (Ch. 8 of this volume), Newton makes the task of answering (1) an empirical matter; I have argued here that he also renders question (2) a nonquestion.[16]

FROM BODIES TO THEIR PARTS

In this section we consider the status of the parts of bodies. The composite systems we have considered above are constructed from bodies, and as such have actual parts. The question we will consider here is whether the bodies themselves have actual parts, and if so, what account we should give of those parts. For this purpose, I will frame the discussion in terms of the actual/potential parts debate. Here, I am deeply indebted to Thomas Holden's book, *The Architecture of Matter*, which is all about the actual/potential parts debate in the seventeenth and eighteenth centuries. The actual parts doctrine (see Holden 2004: 80) states that the parts into which a material body can be metaphysically divided (i.e., the parts into which God could break it, even if no natural process could) are *actual* parts, where *actual* parts are parts that are independent existents that exist prior to any act of division, and are ontologically prior to the whole. Thus, given the actual parts doctrine, bodies are composite entities whose parts have a more fundamental ontological status than the bodies themselves. The potential parts doctrine (see Holden 2004: 79), by contrast, states that the parts into which a material body can be metaphysically divided are *potential* parts, where *potential* parts are merely possible existents until actualized by an act of division. As Holden is at pains to emphasize, a crucial issue in the debate concerns the apparent conflict between the infinite divisibility of matter and the actual parts doctrine: conjoined, these two theses imply that every body is constituted by an actual infinity of parts, and this was held by most of those involved in the debate at the time to be seriously problematic.

According to Holden, both Descartes and Newton are, in different ways, actual parts theorists. He writes:

> The actual parts doctrine is quite orthodox in this dominant tradition in early modern physics and metaphysics, and is ratified by nearly all the new philosophers of this period. . . . First, the doctrine is endorsed by philosophers representative of the two great systems within the new science: the system of Descartes and the Cartesians on the one hand, and the system of Newton and the Newtonians on the other.
>
> (Holden 2004: 86)

The difference between them lies in how they respond to the threat of paradox arising from infinite metaphysical divisibility: according to Holden, while Newton denies infinite metaphysical divisibility, Descartes endorses both the actual parts and the infinite metaphysical divisibility theses while admitting that it is difficult to understand how they fit together.

In the following sections of the chapter, my goal is to do two things. First, I will call into question the claim that Descartes and Newton were actual parts theorists, and I will suggest that neither philosopher's position fits neatly into either the actual or the potential parts camps. Second, I will argue that the law-constitutive approach as applied to the parts of bodies yields an interesting alternative account, and I will suggest that it is one that fits each philosopher much better.

DESCARTES AND THE DOCTRINES OF ACTUAL AND POTENTIAL PARTS

Let us begin by considering the evidence that Descartes subscribes to the actual parts doctrine. First, note that Descartes is clearly committed to the infinite metaphysical divisibility of matter, which he argues for in his rejection of atomism as follows (*Principles* II, art. 20):

> We can also easily understand that it is not possible for any atoms, or parts of matter which are by their own nature indivisible, to exist. The reason is that if there were any such things, they would necessarily have to be extended, no matter how tiny they are imagined to be. We can, therefore, still conceive of them being divided into two or more smaller ones, and thus we know that they are divisible.
>
> (Descartes 1991: 48–49)

Now the question is whether this infinite (or indefinite) divisibility is associated with actual parts, or merely with potential parts. The evidence for the actual parts interpretation offered by Holden relies entirely on Descartes's application of the real distinction to parts of matter. As Holden rightly asserts, Descartes is insistent that the parts of matter are really distinct from one another. For example, in the *Principles* (I, art. 60) Descartes writes:

> For example, from the sole fact that we now have the idea of an extended or corporeal substance (although we do not yet know with certainty that any such substance truly exists), we are however certain that it can exist; and that if it exists, each part of it delimited by our mind is really distinct from the other parts of the same substance.
>
> (Descartes 1991: 27)

However, as Holden remarks, the actual parts interpretation seems to be in conflict with the account of parts of matter that Descartes gives a little later (*Principles* II, art. 25) where he states:

> By *one body*, or *one part of matter*, I here understand everything which is simultaneously transported; even though this may be composed of many parts which have other movements among themselves.
>
> (Descartes 1991: 51)

Holden resolves this apparent conflict by distinguishing between physical bodies (*merely* physically unified beings) and metaphysical "really distinct" individuals. He writes:

> It is true that, for the purposes of his dynamics, Descartes holds that the parts of matter are individuated by their relative motion, such that the rupture and separation of a previously undifferentiated portion of matter creates two distinct bodies from one. . . . And this may seem less like an actual parts account and more like a potential parts analysis where division creates rather than unveils parts. But this account applies merely to dynamics. At the metaphysical level—the level of individuation into "really distinct" substances or independent beings, rather than the merely physically unified beings that concern dynamics—Descartes consistently maintains an actual parts account.
>
> (Holden 2004: 86)

This is Holden's case for Descartes as an actual parts theorist. However, continuing a debate that goes back to Descartes's earliest commentators, the contemporary literature remains divided, both on the issue of whether Descartes endorsed an actual parts metaphysics, and on the relationship between the bodies that are the subject of Descartes's laws and the parts of matter that are the subject of his metaphysics. For example, Normore (2008) endorses the actual parts interpretation, as does Rozemond (2008: 169), who also notes that this interpretation is not uncontroversial, while Lennon (2007) argues that, according to Descartes, the division of extended substance into bodies is mind-dependent. That this is such a thorny area of interpretation suggests that something rather different may be going on, which reconceives the issues not in terms of the traditional actual/potential parts dichotomy.

The lack of consensus in the current literature derives in part from the paucity of quotations in Descartes's corpus directly endorsing either the actual or the potential parts position. Advocates of one or other position attempt to construct an argument that derives their preferred interpretation from premises that Descartes explicitly endorses. It seems to me that the lack of direct evidence is revealing, indicating that we should not try to push Descartes's position into the framework of the actual/potential parts dichotomy. Descartes does not fit neatly into either camp and has, I think, at least the beginnings of a much richer and more original position. As a way to illustrate this suggestion, consider the following passage from the *Principles* that might be taken to support the actual parts interpretation.

Principles II, art. 34 includes in its title that "matter is divisible into an indefinite number of parts."[17] Descartes has been discussing the division of matter into parts by motion, and in this paragraph he argues for "a division of certain parts of matter to infinity"—that is, an *actual* division. The argument considers an ever-restricting neck through which the parts of matter must pass, in making their circular motion (the accompanying diagram is of *non*concentric circles, with the parts of matter setting out from G and heading toward the "neck" at E), and runs as follows:

> For it is not possible for the matter which now fills the space G to fill successively all the spaces of very gradually decreasing size which are between G and E, unless some of those parts adapt their shape and divide as necessary to fit exactly into the innumerable dimensions of those spaces. In order for this to occur, all the particles into which one can imagine such a unit of matter to be divisible, which are truly innumerable, must move slightly with respect to one another; and however slight this movement, it is nevertheless a true division.
>
> (Descartes 1991: 57)

As Roux (2000, esp. pp. 223–230) rightly insists, Descartes is arguing for an actual division of parts to infinity. This is consistent with what

Holden says about Descartes recognizing that his own system must face the challenge posed by actual indefinitely divided matter. However, the above argument does not require that matter *per se* is indefinitely divided. Rather, it requires that there are *some* parts of matter that are actually indefinitely divided, for some periods of time. The paragraph that follows (*Principles* II, art. 35) makes clear that this is Descartes's intention. He writes: "It must be observed that I am not talking here about all matter, but only about some part of it," and goes on to describe larger parts of matter "mingled with" those that are indefinitely divided. Indeed, this coheres well with the various passages where Descartes appeals to the different-sized parts of matter. It seems to me that this passage is one place where Descartes's position can be understood as resisting the potential/actual parts dichotomy.

It is beyond the scope and aims of this chapter to develop a detailed interpretation of Descartes's position along these lines; however, in the final section of this chapter I will argue that the law-constitutive approach gives us a principled way to understand the new approach to the parts of bodies that is, I suggest, embryonic in Descartes.

NEWTON AND THE DOCTRINES OF ACTUAL AND POTENTIAL PARTS

Holden makes frequent claims about Newton being an actual parts theorist, but when it comes to quotations, the evidence is scant, and almost entirely from very early writings. Of the four arguments for the actual parts doctrine that Holden identifies, he finds evidence of only two of them in Newton, both appearing in the so-called *Trinity Notebook*. Indeed, in a footnote Holden himself remarks: "I cannot find an explicit statement of the doctrine in the mature Newton's published writings, though it is strongly suggested in a draft written around the period of the *Principia Mathematica* 2nd edn." His reference is to McGuire 1978: 117, where Newton is discussing the nature of space, and that it has no parts. During this, Newton uses the phrase "nor are there more parts in the totality of space than there are in any place which the very least body of all occupies," and it must be this to which Holden is referring. However, Newton is using this phrase, "least body of all," to illustrate the nature of space; if this is the best evidence that the mature Newton was an actual parts theorist then it is weak evidence indeed.

I suggest that perhaps this is because the dispute becomes a nondispute for Newton, and does so in part because of his law-constitutive approach to bodies (which emerged later than the *Trinity Notebook*; there is no hint of it there). Following the *Principia*, we are hard pressed to find among Newton's metaphysical commitments any that are unrevisable in the light of subsequent developments (arrived at via his maturing methodology for natural philosophy).[18] This is surely true of his atomic hypotheses. Once Newton has developed the law-constitutive approach, the solution to whether there are ultimate bodies becomes dependent on the laws. This point should be clearer after we have considered explicitly the law-constitutive approach to the parts of bodies, to which we now turn.

THE LAW-CONSTITUTIVE APPROACH TO THE PARTS OF BODIES

When discussing the construction of composite systems from bodies above, we saw that, according to the law-constitutive approach, the components of a composite system are those parts of the system that obey the laws. If we now ask of a given body, "what are its component parts?," then the very same law-constitutive analysis of parts can be applied.

I want to stress that this is a metaphysical thesis about the status of parts. Common to the actual and potential parts doctrine is the claim that "a body *is* an aggregate of parts" (be they actual or potential). In the law-constitutive view we say, "a body *is* whatever satisfies the laws," and this is independent of what we may say about actual/potential parts, so we free ourselves from having to make prior metaphysical commitments concerning parts in order to say what a body is. However, the law-constitutive approach *does* make commitments concerning the parts of bodies: to be an actual part of a body is to interact in accordance with the laws. The law-constitutive view denies that *any* old part of a body is also a body, and gives us a rule for telling *which* parts of a body are in fact also bodies. And these bodies are its actual parts.

Let me emphasize that this is not primarily about physical divisibility or about material structure: it is about metaphysical divisibility. Whether a body has actual parts depends on whether it is a composite of parts that themselves satisfy the laws, but such parts need not be *physically* divisible from one another. For example, if the strength of the force dominating the interactions between the parts is great enough, and goes up exponentially with distance, then arguably the parts are not physically divisible from one another, even though they remain metaphysically divisible. As regards material structure, a body might be entirely homogeneous, and yet through how it changes shape over time, or how it moves, might reveal that it has component parts.

My claim is that what is being offered here is an alternative to the dichotomy of actual versus potential parts, and is distinct from these two positions in the following ways. First, the law-constitutive approach to the parts of bodies rejects:

- the actual parts view that *any* part of a body is an actual part;
- the potential parts view that there are *no* actual parts until the body is physically divided into those parts.

Second, the law-constitutive view also rejects:

- the actual parts view that the parts are ontologically prior to the whole;
- the potential parts view that the whole is ontologically prior to the parts.

Elaborating on this second point, the actual parts, as bodies, will have the *same ontological status* as the compound body. A body *may* have actual parts in the law-constitutive view, but those parts (as bodies) will have the *same status* as the compound body has—no body is ontologically parasitical on its parts, and no part is ontologically derivative on the whole. Insofar as the laws contribute to saying what a body *is*, and what a composite system *is*, and what a part of a system *is*, composite systems are just as fundamental as non-composite systems. As a metaphysical thesis, the law-constitutive approach to bodies, component systems, and parts, flattens out the ontological hierarchy.

I offer the law-constitutive approach as a way of thinking both about Descartes's and Newton's own positions on the status of bodies and their parts, and also about how—with hindsight—the actual/potential parts debate came to be dissolved rather than resolved, by the advent of a new position. We are the inheritors of this new position, and it is one to which contemporary metaphysics should pay heed.

In thinking about Descartes's position on the status of bodies and their parts (discussed above), I claimed that for Descartes, all matter is *potentially* divisible *ad infinitum*, that the principle for the *actual* division of matter into parts is motion, and that this actual division is finite in some regions, whereas in others the indefinite divisibility is actualized. This position does not correspond well with either the potential or the actual parts doctrines, but it does fit neatly with the analysis of actual and potential parts that follows from the law-constitutive approach. When it comes to Newton's position, his silence in his mature work on the actual/potential parts issue shouts loudly, to my ears at least, but leaves us with no explicit evidence. Clearly, the issue becomes a nonissue, and my speculation is that this is, at least in part, because of his use of *laws* to guide his search for answers concerning ontology. In particular, Newton is explicit in his law-constitutive approach to bodies, and if he conceives of the parts of bodies as themselves bodies (as perhaps his rules of reasoning might encourage us to believe), then the law-constitutive approach extends to cover the question of divisibility. This is not to say that either Descartes or Newton explicitly advocated such an approach, of course, but it is to claim that—with the benefit of hindsight—we should see in each of Descartes's and Newton's work a crucial chapter in the story of a profound philosophical transformation in the basic framework of matter theory.

CONCLUSIONS

I have sought to challenge the view that Newtonian mechanics deals with "vanishing bodies," treating them as mathematical entities and remaining silent about metaphysical questions concerning their nature. I have argued that if we adopt the law-constitutive approach a different picture emerges, one in which the apparent silence is because matter theory is no longer prior to mechanics, and must be developed in partnership with mechanics. The laws give necessary conditions on *what it is* to be a body, on *what it is* to be a composite system of bodies, and on *what it is* to be a *part* of a body. Traditionally, these questions belong to matter theory, and to metaphysics, but with the development of Newtonian mechanics the two become entangled.

I have offered two examples of the deep matter-theoretic significance of the law-constitutive approach with respect to composite systems. I have argued that if we adopt the law-constitutive approach, Newtonian mechanics provides a principle of unity for composite systems. This principle of unity should be interpreted as a metaphysical principle, providing the necessary and sufficient ground for the composite system to constitute a genuine whole. I have also argued that if we adopt the law-constitutive approach, Newtonian mechanics provides a new position in the actual/potential parts debate, cutting across the traditional dichotomy and offering a new way to approach the question of metaphysical divisibility. Both these aspects of Newtonian mechanics deserve to be treated as philosophically serious contributions to metaphysics, and when this is done they profoundly alter the framework within which discussions of matter theory should take place. I think this shows that Newtonian mechanics is not at all silent when it comes to matter theory.

ACKNOWLEDGMENTS

Material from this chapter was presented at the Bucharest colloquium on “Vanishing bodies and the birth of modern physics” in June–July 2008, and I am grateful to those present for their suggestions. Particular thanks to the editors of this volume for their comments.

APPENDIX: THE CONSTRUCTIONAL STRATEGY IN DETAIL[19]

The Theoretical Solution

It is worth spending some time looking at Newton’s constructional strategy in more detail, to see the way in which the strategy allows one to treat any composite system as a body, and also to provide the parts of the composite systems and treat them also as bodies.

The first use of Newton’s third law in the *Principia* is found in Corollary III to the laws of motion, where Newton uses his third law to demonstrate that the total quantity of motion before and after a collision between two bodies is conserved. He is demonstrating conservation of motion for two colliding bodies, but he is also doing more than this. For Newton, quantity of motion is not Descartes's scalar notion but rather the vectorial concept, momentum. Unlike Descartes's concept, this concept in conjunction with third law allows us to go beyond the claim that the total quantity of motion is conserved to the redistribution of the total quantity of motion, both in terms of the magnitude of the momentum and in terms of the direction of the motion. We have a quantified solution to the distribution problem, and an extension of the conservation of linearity of motion from single bodies to pairs of colliding bodies. Given two bodies individuated via the numbers attaching to certain quantities (mass and velocity) that then collide, we can now re-identify each of them after the collision because we have a rule for how those numbers change for each individual as a result of the collision.

The next challenge is to generalize this to many-bodied systems. In Corollary IV Newton shows that redistribution of motion in interactions by means of his third law is consistent with first law holding for a composite system treated as a single body via the center-of-mass of the system.[20] The structure of Newton's argument is to build up from the behavior of a set of mutually isolated bodies, via a pair of interacting bodies, to a many-bodies system of interacting bodies. In detail, Newton begins with a set of bodies each of which is freely moving and straightforwardly argues that "the common center of gravity of any two either is at rest or moves forward uniformly in a straight line" (Newton 1999: 422). Then, he considers an isolated system of two interacting bodies. Given the second and third laws, any change in the momentum of one body will be accompanied by an equal and opposite change in the momentum of the other, and hence the center-of-mass of the two-body system remains at rest or in uniform motion.[21]

Next, he adds to this pair of interacting bodies the remainder of the set of mutually isolated bodies with which he began. Combining the above results for the set of non-interacting bodies and the pair of interacting bodies, he concludes that the motion of the center-of-mass of the combination will be unaffected by the interaction of the pair.

Finally, we need to extend this to composite systems in which three or more bodies are interacting. Newton says: "Moreover, in such a system all the actions of bodies upon one another either occur between two bodies or are compounded of such actions between two bodies" (Newton 1999: 20). This is the point that is crucial for the problem of individuation of the component bodies of a many-bodies system. It means that the solution given above for a two-body system holds even when we add more bodies to our system; we are still able to use the rules Newton has given us to calculate the numerical change in velocity that an individual body will undergo as a result of a collision with another body.

On the question of the generalized conservation law, from here Newton concludes that: "Therefore, the law is the same for a system of several bodies as for a single body with respect to perseverance in a state of motion or of rest" (Newton 1999: 423). Conservation of linear momentum is shown to hold for a composite isolated system of interacting bodies via redistribution of motion according to his third law, and the method is to generalize by construction from a single isolated body to a composite isolated system. In this way, we see that the new cosmology is built from isolated subsystems that preserve their state unless acted upon by a force, and that preserve their identity when interacting with other systems, by means of conservation principles.

Newton's Constructional Strategy in Practice

This method of building the cosmos is put into practice in Newton's discussion of planetary motion.[22] For example, in discussing the motion of the satellites of planets Newton (*Principia,* Book 3, Proposition XXII, Theorem XVIII) writes that they will move around their planet but that this motion will be disturbed from a perfect ellipse by the influence of the sun. We can construct the actual motion of a planetary satellite by beginning from a consideration of the satellite plus its planet as a two-body composite system isolated from all other influences.

Newton then goes on to describe the way in which the moon deviates from an elliptical orbit of the Earth, and in Proposition XXV of Book 3 (Newton 1999: 839) he shows how to "find the forces of the sun that perturb the motions of the moon" by considering a system consisting of the moon and Earth only, and then analyzing the actual motion of the moon as a deviation from this idealization.

We end by noting one final feature of this constructional strategy. We have seen that according to Newton the behavior of the three-body system can be analyzed in terms of how the two-body system would have behaved plus a disturbing factor. In other words, the interaction between the sun and the Earth is completely blind to whether or not the moon is present. The overall behavior of the Earth results from its own behavior as an isolated system, plus the contribution arising from its interaction with the sun, plus the contribution from its interaction with the moon, and so forth, and each of these contributions is completely unaffected by whether or not the other contributions are present. In this way, we can proceed to reconstruct the entire universe, adding one body at a time, and nothing that we add will ever require us to go back and recalculate how the sun and the Earth interact.

In conclusion, then, at the heart of the Newtonian cosmos of the *Principia* lies Newton's solution to Descartes's problem of the individuation of material bodies, many crucial aspects of which (taking isolated individual bodies as the starting point, the concept of the state of the body specified numerically and without appeal to the "underlying nature of matter," conservation laws, and the constructional strategy) are found also in Descartes's own solution. Newton certainly made important changes in the process of arriving at his solution, but the basic strategy remains the same.

NOTES

1. The material in this section summarizes claims made in Brading forthcoming. Discussion of the "problem of bodies" can be found within the broader context of the debate over the status of the parts of matter in Descartes's metaphysics, to which I will return below (and see below for references to this literature).
2. Descartes's God is so powerful that he could divide matter into parts in ways incomprehensible to us, presumably, but that will not do here because Descartes requires that we clearly and distinctly perceive that matter is so divided. Therefore, on Descartes's own terms, God must be dividing matter into parts in a way that is intelligible to us and can be accounted for within Descartes's metaphysical system. I think that the issue of our clear and distinct perception that matter is so divided poses a *prima facie* challenge to Normore's recent (and intriguing) suggestion that Descartes takes the individuation of the parts of matter as basic (see Normore 2008), and similarly that those who distinguish Descartes's "parts of matter" from the bodies that are the subject of his physics (such as Holden 2004) owe us an account of this division into parts that satisfies the clarity and distinctness requirement.
3. Normore (2008) has recently suggested that there may be a third option: that we take individuation of the parts of matter as basic. He makes a strong case for this suggestion but, as noted above, I would like to know how he responds to the requirement that our perception of the division of extended matter into parts be clear and distinct.
4. The sections addressing (1) draw heavily on joint work with Dana Jalobeanu, friend and long-term collaborator.
5. Jalobeanu (2003) argues for deeper differences between the two projects than has been hitherto acknowledged in the literature.
6. Garber (1992) points out that although in *The World* and the *Principles of Philosophy* the law of conservation of quantity of motion is presented as a special case of the more general principle that a system will conserve its state unless acted upon externally, chronologically Descartes had the special case first and the general case appears for the first time in *The World*.
7. Gaukroger (1995), for example, views both the first and second laws of *The World* as conservation laws.
8. Garber (1992), for example, calls this law, and its development in the *Principles*, the "law of impact."
9. Indeed, having labeled the second law the "law of impact," Garber goes on to criticize the law for failing to solve the problem of collisions, concluding: "Descartes's purported impact law in *The World* is, thus, no impact law at all" (1992: 232). It seems to me that this counts heavily against the "impact" interpretation: there is a natural interpretation that renders the law successful (the conservation law approach) and surely, all things being equal, this is to be preferred over an interpretation whose outcome is that the law is obviously a failure.
10. The eventual target is the indefinitely extended cosmos in which motion is constantly redistributed in accordance with the general principle that the total quantity of motion of the universe as a whole is conserved.
11. Descartes himself never interpreted his law in this way, of course. However, Spinoza's approach suggests that it is not a huge leap to interpreting the conservation law for composite systems as a principle of unity.
12. Gabbey 1980 and Garber 1992 have argued that this version of the law is best viewed *not* as a conservation law, but as a law about collisions based on

the idea that a collision is a contest between the two bodies. Garber writes: ". . . the impact law, law B of *The World*, appears as law 3 of the *Principles*, considerably changed from its initial statement. The contest view, at best implicit in the earlier discussion, becomes the heart of the law, now clearly distinguished from the conservation principle. . . ." (1992: 234–235). For more on this contest view of forces see Gueroult 1980. A better interpretation, in my opinion, is that the law *remains* a conservation law for the composite system as a whole, but the problem of redistribution is now tackled in terms of a contest. Gaukroger 2000 has offered a distinct and powerful interpretation of the approach Descartes takes to the problem of redistribution, arguing that Descartes is using the model of statics, and in particular a balance, to work out the rules of collision. Thus, a lighter body will never raise a heavier body placed at an equal distance from the pivot point. The "balance" account is made even more convincing by the fact that Wren and Huygens both used balance analogies in their attempts to solve the problem of collisions (in response to the Royal Society challenge); see Radelet 2000.

13. The genesis of this solution can be traced in the way Newton's laws develop through earlier manuscripts to their final incarnation in the *Principia*. For discussion of the development of the laws see especially Westfall 1971: 439ff, and Herivel 1965.
14. A case for this view of the role of the third law can also be made by considering the historical process by which Newton came to his third law. As Westfall (1971: 344–347) discusses, it is in Newton's attempt to solve the problem of collisions that he develops his concept of force. In this way we get (a) a measure of the external cause of changes of motion of a body, and (b) the separation of the concept of force from the concept of quantity of motion, and so from Descartes's law of conservation of motion for colliding bodies, giving us Newton's third law as the underpinning of the redistribution of the total quantity of motion (where quantity of motion is now the vector quantity momentum) in a collision, such that momentum is conserved.
15. Question (2) might take the form: in virtue of what is a body is hard (say), such that it can undergo collisions? The Cartesian reductionist seeks to reduce hardness to shape, size, and motion. In this way, hardness can be defined (in terms of shape, size, and motion) prior to its use in giving the laws of collision. However, it was already clear to many that the "strong program" is not going to work, and a weaker version of the project rejects the Cartesian restriction to shape, size, and motion and seeks to identify the appropriate properties to include in the reduction base. Once this move is made, question (2) becomes problematic: it is no longer clear what role an answer to this question has, even if one could be given.
16. It is a nonquestion except insofar as the law-constitutive approach can be brought to bear on it, but this style of tackling the question is not at all what Neile and his contemporaries had in mind: they were seeking a law-independent matter theory.
17. I am grateful to Daniel Garber for drawing my attention to this paragraph and to the article by Sophie Roux cited below.
18. Janiak (2008) argues that Newton rejects action-at-a-distance and that this remains unrevisable.
19. The material in this appendix was developed as part of a joint project with Dana Jalobeanu.
20. Again, there is a long history to this discussion in the *Principia* that can be found in Newton's manuscripts. For discussion of this history see Herivel 1965. Corollary IV reads: "The common center of gravity of two or more bodies does not alter its state of motion or rest by the actions of the bodies

among themselves; and therefore the common center of gravity of all bodies acting upon each other (excluding external actions and impediments) is either at rest, or moves uniformly in a right line).”

21. Or, as Newton writes: “Accordingly, as a result of equal changes in opposite directions in the motions of these bodies, and consequently as a result of the actions of the bodies on each other, the center is neither accelerated nor retarded nor does it undergo any change in its state of motion or of rest” (1999: 423).
22. Cohen (1980: 171–182) discusses this process in detail, and he attributes to the third law the role of allowing Newton to move from consideration of the motion of a single planet about a fixed center of force, to a pair of interacting planets, to a many-bodies interacting system, thereby constructing the motions of the planets in the manner we have described.

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