**Emilie Du Châtelet, *Foundations of Physics*, 1740.**
**Chapter 20. Of dead forces, or pressing forces, and the equilibrium of powers.**
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Footnotes are ours except where otherwise indicated.
Du Châtelet's marginal notes are placed in **{bold}** in the closest appropriate place in the text. Please see the French original for the position of each note in the margin alongside the paragraph.

## Chapter 20: Of dead forces, or pressing forces, and the equilibrium of powers

**518.** Motive Force, which is the principle of motion, makes bodies traverse a certain distance or makes them displace a certain number of obstacles, when it is not prevented from acting, depending on whether it is exerted more or less. But when its action is prevented by some invincible obstacle, then the force does not make the body upon which it acts traverse any space, but makes it strive; the force impresses upon the body a tendency to displace that obstacle, and to impress a motion upon it.

**519. There are two kinds of forces; how to distinguish them.**
We distinguish these two Forces by the terms *Dead Force*, or *Virtual Force*, and *Living Force*. *Dead Force* consists in a simple tendency to motion: for example, the tendency of a spring when it is ready to uncoil. *Living Force* is the tendency a body has when it is actually in motion.

**520.** Dead Forces are also known as *Pressing Forces*, because they press upon the bodies that resist them, and strive to displace them.

**521.** Pressing Forces can either remain at rest with the bodies they press upon, or indeed traverse with them a certain space.

**522.** The Pressing Forces that remain at rest with the bodies they act upon are:
1. The weight of bodies, that which carries them toward the center of the earth; it is by this force that all bodies press upon the obstacle that supports them.
2. The effort a compressed spring makes to uncoil, and to repel the powers that restrain it.
3. The cohesion and the magnetic force by which two bodies press mutually upon one another, rather as our hands press one on the other when we shake hands.

**523.** The Pressing Forces that, in remaining applied to the body upon which they act, move with it, are:
1. The weight that is in the pan of a balance, and that forces this pan to descend with it.
2. A spring that uncoils, and pushes ahead of it the obstacles that were restraining it.
3. My hand that presses upon a body placed on a table, and that traverses the table with it.
4. A body attached to another body with which it rotates, and that it pulls by its centrifugal force, etc.

**524.** Thus, we use the name *Pressing Force* for both the force by which one body pulls another and that by which it presses upon another; in a word, everything that strives to displace a body, whether it is in immediate contact with that body, like the weight in the pan of a balance; or acts upon it through another body, like the rotating body that pulls the one to which it is attached by cord; or, finally, whether it simply presses upon it, like a stone placed on a table.

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**525.** All Motive Force produces pressure; but the pressure of dead force is destroyed at every instant, and that of living force is not.

**526.** The obstacles upon which pressing forces act can be either invincible, or capable of giving way.

**527.** When the obstacles are invincible, the action of the force that strives to displace them is destroyed at every instant by these obstacles, and reproduced at every instant through the continual striving of the pressing force to overcome this resistance. Thus, the small degrees that the pressing force impresses upon the obstacle that resists its action, perish as they are born, and are born as they perish; and it is in this constant reciprocation, in this cycle of creation and destruction, that consists the effect of the heaviness of a body, when it is held back by an invincible obstacle. It is this pressure, destroyed as soon as it is produced, that is called *dead force*.

**528.** Although dead forces produce no effects, they can nevertheless be considered as either active or passive.

**529. {In what dead forces consist.}** The Dead Force that I consider as active is the force that bodies have by which to keep a power in equilibrium.

**530.** The Dead Force that I consider as passive is that which a motionless body receives when it is solicited to move but nevertheless remains at rest.

**531. {What their effect is.}** When dead force is destroyed by an invincible obstacle, its effect is the same whether its action lasts only an instant, or is continued for millions of years. For in both cases the action does not produce any real effect; but only tends, at each instant, to produce one. Thus, however long the pressure against an invincible obstacle may be continued, the force that produces it is never exhausted.

**532.** As soon as the action of Dead Force on an invincible body ceases, its effect, which is the pressure of the body that resists it, ceases also, and its effect never survives its action.

All pressure is consumed while it acts, and its effect in one moment does not depend at all on its effect in another, such that it is always destroyed in an infinitely small instant, whether by the opposing pressure of an invincible obstacle, or in the communicating or the destroying of the force.

We call resistance that which destroys pressure, and so reaction is always equal to action, meaning only that the resistance is equal to the pressure it destroys.

**533.** An obstacle that is invincible for one force is not for another, if the latter force is greater than the first.

**534. {When the obstacle yields, pressing forces or *dead forces* become living forces.}**
When the obstacles upon which motive force acts are not invincible, the action of this force upon these obstacles is to make them leave their place; and then the small degrees of motion that this force communicates, at each infinitely small instant, to the body upon which it acts, accumulate and are conserved therein, and this force compels the body to change place: in this case *dead force* changes into *living force*.

**535.** It can already be seen that dead force and living force differ essentially from one another, since the former does not produce any effect, and the latter produces a real effect, which is the displacement of the obstacle. Thus, the two kinds of force can no more be compared than can a line and a surface: they are heterogeneous quantities, and they differ infinitely.

I will discuss living forces in Chapter 21. Here I examine only the effects of simple pressure.

**536. {How dead forces must be estimated.}** In bodies at rest one estimates the force that they have for holding a given power in equilibrium by the product of their mass (or of their proper matter) multiplied by their virtual or elementary speed (that is to say, by the initial speed that they would have were that power, which they retain, in fact to give rise to motion).

**537.** A body takes time to acquire motive force; for every effect presupposes a time in which it operates.

**538.** The power that acts upon the body, and that communicates motive force to it, remains applied to this body until it [i.e. the body] has acquired the force that the power communicates to it.

Motive power remains applied to the body, and traverses with it a certain space in the initial instant in which the power transports the obstacle.

**539.** In the first instant in which the motive power remains applied to the body upon which it acts, the intensity of this power is the product of the mass by the initial speed. For, while the body that is being pressed has not yet acquired all its motion, the power that communicates motion to it is a dead force.

Powers can differ from one another with respect to the size of the masses they can transport, and with respect to the infinitely small space they can traverse with the transported masses in equal times. This is what we call the *intensity of powers*.

**540. {Of the comparison of powers.}** One cannot know the magnitude of a single power; one must compare the momentary action of two powers that act on equal or unequal masses, and that push them with a greater or lesser increment of speed, in order to be able to know in what ratio these powers act; for all our knowledge is only comparative.

**541.** If in an equal space powers displace unequal masses, the intensities of these powers will be as the displaced masses multiplied by their initial speeds.

**542.** If the displaced masses are equal and the spaces unequal, the intensities will be as the spaces.

**543.** If the masses and the spaces are unequal, the intensities of the powers will be as these masses and these spaces; that is to say, in a ratio composed of the two.

**544.** The displaced masses are always in direct ratio to the magnitude of the powers, and in inverse ratio to the spaces.

**545.** Thus, the intensities of the powers are equal if the spaces traversed are in reciprocal ratio to the displaced masses. For example, if the displaced masses are 8 and 6 and the traversed spaces 3 and 4 respectively, the intensity of each of the two powers will be 24, for in this case the first mass is to the second as the initial speed of the second is to the initial speed of the first, and thus the product of the spaces traversed and of the displaced masses, multiplied by one another, represents the intensity of the

powers that communicate the motive force.

**546.** Equal powers acting in directly opposing directions amount to insurmountable obstacles for one another, and mutually destroy each other's effects. Thus every opposing power can be considered as an invincible obstacle in relation to the power that it counterbalances; and every invincible obstacle can be considered as a power equal to the power of which it has prevented the effect.

**547.** When powers are in equilibrium, the dead forces are in a compound ratio of the masses and their virtual speeds.

**{Why 2 pounds and 10 pounds appear to be in equilibrium.}**

Thus, when 10 pounds appear to be in equilibrium with 2 pounds, as in a steelyard balance, this is in fact only an illusion; for it is not between 2 and 10 that there is equilibrium, but between 2 and 10 arranged such that the 2 pounds would have five times more speed than the 10 were they to move, which reestablishes the equilibrium.

Equilibrium is therefore rest caused by the opposition and equality of two or more forces.

**548.** Two forces can be in equilibrium, and mutually destroy one other, only when they would make the same mass traverse equal spaces in equal times, were the mass to yield to their action in the first moment. For these forces could displace the same masses even though they were not able to transport them equally far in equal times; if one of these powers, for example, could make the same body traverse an infinitely small space, double the infinitely small space that the other power would make it traverse in the same time, then the intensity of this power would be double that of the other power, for when the masses are equal, the powers are as the spaces (§542).

**549.** Equal and opposite powers mutually destroy one another, and so their destruction is the only effect that they produce.

When two powers are in equilibrium, they are equal.

**550. {Of the equilibrium of powers.}**

In order for two powers to be in equilibrium, their directions must meet at a point and converge along the same line, without which they would not be opposed, or indeed they would be so only partly.

**551.** If two powers act on the same body in contrary directions and with unequal forces, the force of the weaker power will be destroyed, as will an equal part of the force of the stronger power, such that the stronger power will push the weaker in front of it with the force that remains to it; and the effect produced will be equal to the force remaining to the stronger power.

**552.** If an obstacle is on the paths of two equal powers moving in opposite directions, neither these powers nor the obstacle will be displaced; and these powers will mutually destroy one another's effects, so long as they continue to press on this obstacle in opposite directions.

**553. {Plate 11. Fig. 69} {In what proportion powers in equilibrium must be to one another.}**

In order for three powers A, B, C, whose directions meet at point D, to be in equilibrium, their respective intensities must be as the three lines DG, GE, ED, parallel to the directions of the three powers A, B, C, which together form the triangle DGE or DEF. For if power B pulling from point D had given it speed DG, and if power C had given it speed DF=GE, point D would have traversed the diagonal DE of the parallelogram GDFE.

Therefore, in order for power A to keep point D at rest, and counterbalance powers C and B, it must be able to give to body B speed ED; for then the force towards DE will be equal to the two forces towards DG and towards DF=GE, since forces are among themselves as the speeds that they communicate to the same body (§257). The sides of triangle DGE therefore express the ratio for these three powers to stand in equilibrium among themselves.

**554.** A power is in equilibrium with 4, 5 or any number of powers when all the powers that counterbalance it can be contained in a single power, the intensity of which is equal to the counterbalanced power, and if moveover they act together along the same line as that power.

**{Fig. 70}** Let this point A be pulled by the five powers D, E, F, G, B so that the power B is in equilibrium with the four other powers D, E, F, G. If these five powers are respectively proportional to the lines AD, AE, AF, AG, AB then, once we have formed the triangle ADC or the parallelogram ADCE, powers AE and AD will be contained in the single power AC that acts in direction AC; thus powers AD, AE, and AC will be in equilibrium (as shown in the preceding section).

**{The action of any power can be resolved into two other powers.}** Powers AG, AF being then contained in the same way in power Ah, these two new powers AC and Ah will be reduced in the same fashion to the single power Ab, which will be equal and directly opposite to AB, since it will be along the same line and represent forces AE, AD, AF, and AG which were in equilibrium with AB.

**555.** It follows from §553 that the action of any power can be resolved into the action of two or more powers, and this can be done in an infinity of different ways, because of the infinite number of triangles that can have the same side (§281).

Thus, we can consider the effect brought about by several powers as being the effect of a single force that is equal to them, and vice versa.

**556. {Proof of the equality of action and reaction by the equilibrium of powers.}** It is above all in the combination of the action of pressing forces that one finds the fulfilment of the third Law of motion, by which the reaction is always equal to the action (§258) for pressing forces never act without there being an equal resistance, whether the obstacle yields or resists invincibly.

Thus, when two are more powers are in equilibrium, even if they press upon one another and if the smallest increase in force would displace them, nevertheless they all remain in place, so long as their mutually opposing efforts are equal.